

STUDENT VERSION

CATAPULT LAUNCH - UP AND AWAY

STATEMENT

1. A projectile is fired from a catapult. The angle of elevation above the horizontal for the launch is α (in radians or degrees). For a given speed v_0 the initial velocity is $v(\vec{0}) = \langle v_0 \cos(\alpha), v_0 \sin(\alpha) \rangle$. You should verify this. The initial launch position is (x_0, y_0) .

We use g as the acceleration due to gravity (either 32 ft/sec^2 or 9.81 m/sec^2). The functions $x(t)$ (down range) and $y(t)$ (height) describe the coordinates of the projectile at time t in seconds. Let us not take into consideration air resistance. If we did how would the trajectories change?

The distance that a catapult fires a projectile depends on two factors: (1) the projectile's initial velocity v_0 and (2) the angle of elevation, α , of the launch of the projectile from horizontal. Suppose you can give the projectile an initial velocity of $v_0 = 300 \text{ ft/sec}$.

Investigate the effects of varying the angle of elevation from the horizontal on the horizontal range of the projectile. What angle maximizes this range?

2. You are at the base of a long, steep hill which has an angle of elevation from the horizontal measuring 60° . Before you lies a level plane. A target on the plane lies 3,000 ft from where you stand. You decide to catapult a projectile set at 45° angle of elevation of the launch of the projectile from horizontal. (See (1) above for your own confirmation that this angle is best.) Your projectile has an initial velocity of 300 ft/sec.

Because of a stream which runs across the plane, you can only advance the catapult 30 ft. Where should you put it in order to score a direct hit on the target?

3. How far can we put the target and have it still within range of our catapult? Justify your answer.