Biology-Driven Approaches to Teaching Differential Equations

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Abstract

We present SIMIODE - Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations, a community of teachers and students committed to teaching differential equations in a modeling first approach. Of all the areas that mathematics embraces as it reaches out to show students applications of their mathematics, biology stands out as a major discipline. We demonstrate several examples, of many points of contact, in which mathematical modeling in biology provides the motivation for the study of selected topics in differential equations.

Keywords: biology, differential equations, modeling, motivation

1 Introduction to SIMIODE

SIMIODE - Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations is about teaching differential equations using modeling and technology up front and throughout the learning process. The SIMIODE community, at www.simiode.org, is where colleagues and students can communicate, share, contribute, collaborate, publish, teach, and explore, in support of modeling first teaching of differential equations in context.

Creating a community of learners embracing interdisciplinary approaches to mathematics [8] is appropriate for all disciplines and for teachers and students. SIMIODE is building a complete environment for teachers and learners, including communication, groups across and intra/inter campus projects for students and teachers, models, data, videos, and more, e.g., SIMIODE YouTube videos at [5] where students can collect data on Torricelli's Law for a falling column of water and model it with a differential equation.

2 Biological Links in SIMIODE

SIMIODE offers a growing number of modeling scenarios which reach out to biology in support of teaching differential equations. These include, as examples, studying absorption of a vitreous bubble of gas inserted after retinal surgery, compartment models of LSD and arithmetic test degradation[6], spread of a disease in a boarding school[4], kinetics reaction studies, spread of ICU's in hospitals, ants building tunnels, effects of ibuprofren for patients on different diets, dialysis machine modeling, insect colony optimization, evaporation of alcohol, lipoprotein metabolism, creatinine kinetics in dogs, paracetamol absorption, time of death, using tea leaves to adsorb heavy metals from water, population studies, and sublimation of dry ice. SIMIODE offers a light approach for starting a differential course uses M&M's[7] to simulate and collect data for death and immigration with parameter estimation as well as equilibrium and stability topics, all brought out at the very first class of the course.

We examine two biology-based modeling scenarios from SIMIODE in support of teaching differential equations and interesting students in the symbiosis between biology and mathematics in teaching differential equations in context in a modeling first approach.

2.1 LSD Absorption

Five male volunteers were administered LSD and then tested on performance on simple addition questions. Both performance score and plasma concentrations of LSD were recorded at 7 times after the initial infusion of LSD.[1, 3]

The student modeling activity in SIMIODE is, "Build a mathematical model which will predict successfully the levels of LSD in the bloodstream and plasma and then correlate these levels with the performance data on the arithmetic problems given to the subjects. Be sure to give crisp assumptions and define all variables, units, and relationships clearly and render a connected analysis. Then summarize your results."

Students build a two compartment model for LSD flow between plasma and tissue compartments in the human body where the volume (in milliliters) of the plasma (V_P) is 16.3% of the volume or mass (M) of the body and the volume of the tissue (V_T) portion of the body is 11.5% of the volume or mass of the body. Concentrations of LSD in plasma and tissue are given by $C_P(t)$ and $C_V(t)$, respectively.

$$\begin{aligned} V_P C_P'(t) &= k_a V_T C_T(t) - k_b V_P C_P(t) - k_e V_P C_P(t) \\ V_T C_T'(t) &= k_b V_P C_P(t) - k_a V_T C_T(t)\,. \end{aligned} \tag{1}$$

Rate parameters k_a, k_b, and k_e, are estimated by minimizing the sum of square errors (2) between model $C_P(t_i)$ and observation data O_i

$$SSE(k_a, k_b, k_e) = \sum_{i=1}^{7} (C_P(t_i) - O_i)^2\,. \tag{2}$$

These obtained parameters verify the model of LSD in the plasma as shown in Figure 1.

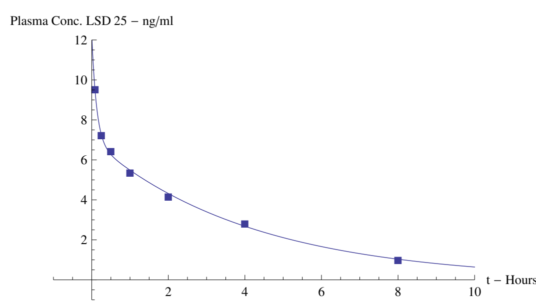

Figure 1: Plot of observed values of LSD (ng/ml) (squares) in plasma and model.

2.2 Boarding School Epidemic

Historical data on an English boarding school epidemic[2] has been offered in an article[4] and in a SIMIODE modeling scenario. Students readily build a SIR model for the epidemic where $S(t)$ is the number of students susceptible to the disease at time t, $I(t)$ includes all students infected or infectious at time t, and $R(t)$ is the number of students who have recovered at time t. Using Excel's Solver capabilities and simple Euler method solution for

(3) students can estimate the parameters a and b in the model and validate their model with the data (Figure 2).

$$\begin{aligned} S'(t) &= -aS(t)I(t) \\ I'(t) &= aS(t)I(t) - bI(t) \\ R'(t) &= bI(t). \end{aligned} \tag{3}$$

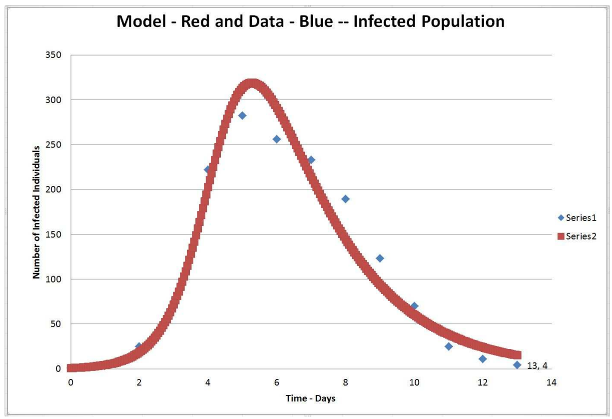

Figure 2: Model of number of infected patients at the English boarding school plotted over the actual data, using (1) with model parameters, $a = 0.002661$ and $b = 0.4944$, determined by Excel's Solver command which minimizes the sum of square errors between the model and the data.

Conclusion

We have offered two examples of mathematical modeling in biology which demonstrate how students can make the link between biology and mathematics while learning new mathematics in context; in this case differential equations. There are many more we will share.

References

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