STUDENT VERSION
Predator-Prey: Blue Whales and Krill

Lydia Kennedy
Mathematics Department
Virginia Wesleyan College
Norfolk VA 23502 USA

STATEMENT

Throughout this handout, there are activities to be done and questions to be answered. In a separate document (Microsoft Word, LaTeX, etc), answer the questions and include charts where indicated. You must also submit an electronic copy of your Excel spreadsheet at the end of the project.

One of the favorite foods of the blue whale is krill. Blue whales are baleen whales and feed almost exclusively on krill. These tiny shrimp-like creatures are devoured in massive amounts to provide the principal food source for the huge whales. In the absence of predators, in uncrowded conditions, the krill population density grows at a rate of 25% per year. The presence of 500 tons/acre of krill increases the blue whale population growth rate by 2% per year, and the presence of 150,000 blue whales decreases krill growth rate by 10% per year. The population of blue whales decreases at a rate of 5% per year in the absence of krill.

These assumptions yield a pair of differential equations (a Lotka-Volterra model) that describe the population of the blue whales (B) and the krill population density (K) over time given by

$$\begin{aligned} \frac{dB}{dt} &= -0.05B + \left(\frac{0.02}{500}\right)BK \\ \frac{dK}{dt} &= 0.25K - \left(\frac{0.10}{150000}\right)BK \end{aligned}$$

1. What are the units for $\frac{dB}{dt}$? for $\frac{dK}{dt}$?
2. There are two terms on the right hand side in each of the differential equations given above. Explain (with a complete sentence) what each of the four terms relates to in relation to the description given above. Include a reason for why the term is positive or negative.

3. Give an explanation of the terms $\frac{0.02}{500}K$ and $-\frac{0.10}{150000}B$ as they relate to the model above.

We can estimate the population and population density, respectively, of the two species with a simple simulation method that uses the differential equations to find the change in the function values for a fixed time period (1 year in this case). This means that starting with an initial population for the blue whales (B_0) and an initial population density for the krill (K_0), we can estimate the population of blue whales a year later using $B_1 = B_0 + \left.\frac{dB}{dt}\right|_{B=B_0,K=_0}$ and the population density of krill a year later using $K_1 = K_0 + \left.\frac{dK}{dt}\right|_{B=B_0,K=K_0}$.

For example, if $B_0 = 10,000$ and $K_0 = 200$, then

$$\begin{aligned} B_1 &= B_0 + \left.\frac{dB}{dt}\right|_{B=B_0,K=K_0} \\ &= 10,000 + (-0.05 * 10,000) + \left(\frac{0.02}{500}\right) * 10,000 * 200 \\ &= 9580 \end{aligned}$$

and

$$\begin{aligned} K_1 &= K_0 + \left.\frac{dK}{dt}\right|_{B=B_0,K=K_0} \\ &= 200 + 0.25 * 200 - \left(\frac{0.10}{150,000}\right) * 10,000 * 200 \\ &= 249 \end{aligned}$$

In a similar manner, $B_2 = B_1 + \left.\frac{dB}{dt}\right|_{B=B_1,K=K_1}$ and $K_2 = K_1 + \left.\frac{dK}{dt}\right|_{B=B_1,K=K_1}$, etc.

4. Using the simple simulation method demonstrated above, fill in the following table.

Time (in years)	Blue Whale population	Krill population (in tons per acre)
0	75,000	150
1		
2		
3		
4		
5		

5. This sort of simulation is best done with the aid of technology. Use an Excel spreadsheet to
   (a) estimate the population of the blue whales and the population density of the krill each year for 100 years, assuming an initial population density of 150 tons per acre of krill and a population of 75,000 blue whales, and
   (b) make a chart (in Excel) that plots the krill population density versus the blue whale population over those 100 years.

   As part of your answer, give the population of whales and population density of krill (complete sentences!) and a copy of the chart from your spreadsheet.
6. Using the simulation results in your Excel spreadsheet, what do you notice about the population density of the krill and the population of blue whales over the 100 years? If one population or the other becomes very endangered (very low population levels), give a plausible explanation as to why. If one or both populations becomes very endangered, give an estimate of how long it takes for this to happen. (Write complete sentences).
7. What happens when the initial conditions are changed?
   (a) Set the initial population density of the krill to be 300 tons per acre and the population of blue whales to be 75,000 blue whales. Before using Excel to do the simulation, make a prediction about the long-term results for the krill population density and the blue whale population over 100 years. Then, do the simulation in Excel answer the same questions as in #5 and #6.
   (b) Set the initial population density of the krill to be 300 tons per acre and the population of blue whales to be 150,000 blue whales. Before using Excel to do the simulation, make a prediction about the long-term results for the krill population density and the blue whale population over 100 years. Then, do the simulation in Excel answer the same questions as in #5 and #6.
   (c) Set the initial population density of the krill to be 300 tons per acre and the population of blue whales to be 175,000 blue whales. Before using Excel to do the simulation, make a prediction about the long-term results for the krill population density and the blue whale population over 100 years. Then, do the simulation in Excel answer the same questions as in #5 and #6.
8. Now, change the initial population density of the krill and the population of blue whales again. Pick different values and experiment. See what happens. Answer the same questions as in #5 and #6 for at least 3 different sets of initial values. Do not use the same initial values as in #7.
9. Describe what this model seems to say about the interacting species.
10. What factors that influence the population density of krill and the population of blue whales are included in the model? What factors are ignored?
11. Make an improvement in the model to overcome a simplification. Make this improvement in both the written description of the model and add its mathematical counterpart to the model.