STUDENT VERSION
MODELING IED BLASTS

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STATEMENT WITH COMMENTS

In many courses (including ours) the skills that are the easiest to assess form the majority of the course points, even if harder to assess tasks are equally as important. In our modeling with differential equations course, this meant that most of the course-wide graded events were weighted heavily towards computations (solving ODEs) and generally did not include modeling questions that required a student to transform a real world scenario into a math model. Our primary means of challenging students' ability to construct a model were two course-wide projects. However, we decided that the students needed additional experience with this transform process, see Figure 1, so we decided to focus six homework assignments on the construction of a model, or the "Transformation" of a real world scenario into a mathematical model.

The first two exercises here are two of those assignments (the others came during first order ODEs and systems of ODEs). We assigned these exercises with the idea for a solution in mind, but without a proscribed answer. For the homeworks that preceded the two assignments included here, we provided a rough sense of structuring an answer, and feedback on what a good student solution looked like that gave the students a starting point for each of these scenarios.The requirement to students was for them to clearly communicate their model, with an emphasis on the Transform step in Figure 1, in one page.

The third assignment included here is a follow-up that we designed for a course-wide "Applications Day" exercise following a block of lessons on Laplace Transforms. It is much more computational, with a preparation requirement for the students that emphasized the Transform step.

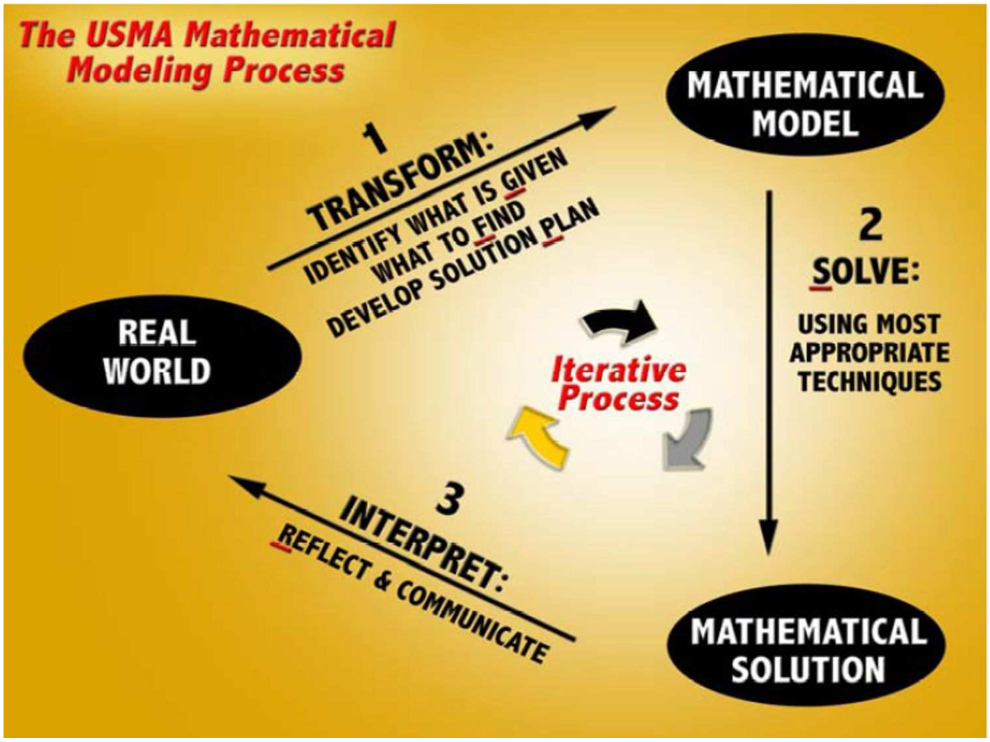


Figure 1. Location of the Transform Step in the Math Modeling Process.

STATEMENT

This is a series of three exercises designed to give you an opportunity for modeling with second order differential equations. The theme of the three assignments is an Improvised Explosive Device (IED) blast on a military vehicle. As a reminder, the goal is to focus primarily on the "Transform" step from the Modeling Process, see Figure 1.

AN IED BLAST

Instructions

This is an individual assignment. You will submit a hard copy report that should not exceed one (1) page, which includes any tables or graphs that you use. Doing the mathematics correctly is important, but it is also critical to be able to analyze and effectively communicate your mathematical results, as well as reflect on their relevance in the real world.

Scenario

Your career as an intelligence officer is off to a fast start with a deployment to the border between Egypt and Israel with Task Force Sinai. Recently, the threat level in the area has increased with the discovery of a handful of emplaced Improvised Explosive Devices (IEDs). There is a lot of

Figure 2. An MATV with Mineroller. Photo from: `http://www.defensemedianetwork.com/stories/u-s-army-and-marine-corps-look-to-enhance-mine-clearance-capabilities/`.

concern about how powerful the IEDs are, and the command is evaluating measures to increase troop protection levels. Recently, every patrol along the border was directed to include a lead vehicle each equipped with a mine roller to counter pressure-detonated IEDs. As you sip your morning coffee, the Task Force executive officer bursts in and says,

> "An IED just detonated on this morning's patrol. Everyone is safe, but the mine roller was blown off the front of the vehicle and up into the air by a buried IED that detonated as the mine roller rolled across it. Luckily, we have the blast on a grainy video from a nearby surveillance system. From that video, we computed how high in the air the mine roller went after the blast, but that's all we know. You need to figure out how to determine the force of the IED blast. We've got to get some sense of how big these IEDs are, and what will happen if one detonates on a smaller vehicle."

You know that you can develop a differential equation model where the dependent variable is the vertical distance traveled by the mine roller. There are a lot of factors involved: the mass of the roller, the attachment mechanism of the roller to the vehicle, the duration of the blast, and more.

Help Your Unit

Develop a model for the distance traveled by the mine roller. Specifically, pay particular attention to any assumptions that you need to make and clearly articulate your reasoning for why your model represents the situation. In the Solve step, identify all appropriate solution techniques for your model, but do not make any computations. For the Interpret step, you will need to describe the approach you will use to solve for the blast force of the IED.

AN IED BLAST - IMPROVED SEATING

Instructions

This is an individual assignment. You will submit a hard copy report that should not exceed one (1) page, which includes any tables or graphs that you use. Doing the mathematics correctly is important, but it is also critical to be able to analyze and effectively communicate your mathematical results, as well as reflect on their relevance in the real world.

Scenario

The threat level in the Sinai remains elevated and your intelligence cell continues to monitor possible IED emplacements by Islamic State affiliated militants. Based on your previous analysis, you know that the IEDs are powerful, and your cell has a good model for the likely size of future IEDs. Task Force Sinai is an international organization, and the many different nationalities involved each bring their own military equipment. The US soldiers operate in Mine Resistant Ambush Protected (MRAP) vehicles which were manufactured specifically for the threats in Iraq and Afghanistan, but many of the other countries operate in older equipment. As you sip your afternoon coffee, the Task Force executive officer bursts in and says,

> "Another IED just detonated on a patrol. Luckily the MRAP absorbed most of the blast, and the soldiers only have minor injuries. While reviewing the damaged vehicle the commander inspected the blast mitigation seating. He's convinced that it made a difference and he's concerned that any vehicle without it is putting soldiers at a greater risk. Come up with an answer for how important these new seats are."

You know that all variants of the MRAP were outfitted with blast resistant seats and that before that change, most US vehicles (the HMMWV for example) did not include blast resistant seats. Blast resistant seats effectively have a shock system built in that can prevent the sudden change in acceleration that can cause Traumatic Brain Injury.

Help Your Unit

Develop a model for the impact of the blast on a soldier in an MRAP.

Figure 3. An example of a blast resistant seat. Photo from: `http://dev.defense-update.com/page/265?attachment_id`.

Specifically: In the Transform step, pay particular attention to any assumptions that you need to make and clearly articulate your reasoning for why your model represents the situation (define your variables and parameters!). In the Solve step, identify all appropriate solution techniques for your model, but do not make any computations. For the Interpret step, you will need to describe the approach you will use to compare the utility of the blast resistant seating to the seating in a normal vehicle without it.

AN IED BLAST - HEAD TRAUMA

This read-ahead is for an INDIVIDUAL in-class problem solving exercise. You are free to discuss it ahead of time. You will be able to use this read ahead, a blank Mathematica notebook, your TI-30 calculator and one 8.5” × 11” hand written, reference sheet along with a Table of Laplace Transforms that will be provided by your instructor. You will use the analytical and technical skills from Block 2 (second order ODEs, including Laplace Transforms) to complete the exercise. The structure of this event mirrors the final course-wide graded event (Application Day 3).

Goals:

1. Students use second order ODEs to model and gain information about real world problems.
2. Students apply a variety of different modeling approaches to the same scenario.
3. Students apply technology learned in the course to analyze and solve a differential equation model and develop an appreciation for how technology enhances their problem solving capabilities.

Technology Skills:

1. Solve a second order ODE with a step function using *UnitStep* and *DSolve* in *Mathematica*.
2. Plot the solutions to an ODE using *Plot* and the replacement command in *Mathematica*.
3. Numerically integrate a function using *NIntegrate* in *Mathematica*.

Modeling Scenario:

You recently arrived at your first battalion, and are assigned as the assistant operations officer, A/S3, while you wait for a platoon leader position. Your unit is now deployed in support of peace keeping operations in the Sinai Peninsula as part of Multinational Force and Observers. Militants in the area recently began emplacing IED ambushes that are severely damaging some of the unarmored military vehicles used by other nations. Because of this threat, your commander has directed the US to assume the bulk of the patrolling duties. Based on the analysis of an IED recovered by Explosive Ordnance Disposal (EOD), your commander thinks it is unlikely that any of the armored US vehicles will be severely damaged by IEDs of the current size, but he is concerned about mild Traumatic Brain Injury (mTBI) as the result of severe acceleration and deceleration on the heads of soldiers in a blast. You have been tasked to work with your battalion medical officer and intelligence officer to decide on the likelihood of mTBI from the current enemy IEDs.

You know that US forces are using armored vehicles that include blast attenuating seats, which is where each seat includes a spring and damper designed to minimize the amount of external force that is applied to a soldier inside the vehicle. The EOD team reports that the recovered IED can produce a force of 100 $[\frac{kg*cm}{ms^2}]$ for 20 milliseconds $[ms]$.

The Metric:

The Head Injury Criterion (HIC) 1 provides a metric for analyzing the likelihood of an acceleration-deceleration head injury based on the acceleration that the head undergoes in a short amount time. For additional references on the HIC, see `http://www.nhtsa.gov/DOT/NHTSA/NRD/Multimedia/PDFs/Crashworthiness/Air%20Bags/finalrule_all.pdf` and `https://www3.nd.edu/~cwample1/Preprints/Head%20injury%20criterion.pdf`. There are many factors that affect the likelihood of internal brain damage, but this is a proven and used metric that serves as a useful starting point. A HIC of greater than 600 means that a person is at risk for a head injury.

$$HIC = \frac{15}{1000} * \left[\frac{1}{15}\int_{T}^{T+15} |a(t)|dt\right]^{2.5}. \tag{1}$$

In this metric, time is measured in milliseconds (ms), $a(t)$ is the acceleration function for the head in gravity equivalents ("in g's"), and T is a time within 15 ms of the maximum acceleration, which in this scenario would be the time of the blast. To effectively use this criteria, you will need to compute the acceleration function for a soldier immediately after an IED blast. Since you can't just write down the acceleration function, $a(t)$, explicitly (because you have no basis for assuming a functional form) consider modeling using a free body diagram.

The Model Basis:

Consider the sum of the forces acting on the soldier, Newton's second law of motion.

$$\text{Force}_{\text{soldier}} = mx'' = \sum \text{Forces}\,.$$

For example, to develop a model for what is happening while during the descent of a paratrooper, where the dependent variable x is the vertical displacement, we can write:

$$mx'' = \sum \text{Forces} = mg - \gamma x'$$

In this equation m is mass, g is gravity, and γ is the drag coefficient. We now have a model that describes acceleration (x''). To find a usable $a(t)$ for our metric, where acceleration is an explicit function of time, we need to solve our model for $x(t)$ using our differential equation solving skills, and then differentiate $x(t)$ twice to get $a(t)$. Then, we will have a function of acceleration that is only in terms of time, t. We can now use that function with the HIC metric to determine the likelihood of mild Traumatic Brain Injury (mTBI).

Mission:

Create a second order ODE that models the situation of a soldier in a vehicle struck by one IED using the sum of forces as the governing principle behind your model. The dependent variable is the vertical displacement of the soldier, measured in cm, and the independent variable is time, measured in milliseconds. Come to class with both a free body diagram and a second order ODE, ready to apply a solution technique and conduct analysis. Be prepared to explain to your classmates how you modeled the blast.

Instructions You have the entire class to complete this exercise. Your computer with a blank *Mathematica* file, issued TI-30 Calculator, Read Ahead (electronic or hard copy), and one 8.5” by 11” handwritten reference sheet (front and back) are the only references authorized (no internet/notebooks/textbooks). Show all of your work.

1 Basic Understanding

Using information provided in the read ahead, do the following:

1.1 Describe why a step function is a useful tool for modeling an IED blast.

1.2 What is the purpose of a blast attenuating seat and how does it relate to what we have learned about spring-mass systems?

1.3 If my injury threshold is a function of acceleration and I know something about the forces acting on the soldier, why do I need to solve the differential equation?]

2 Solve the ODE

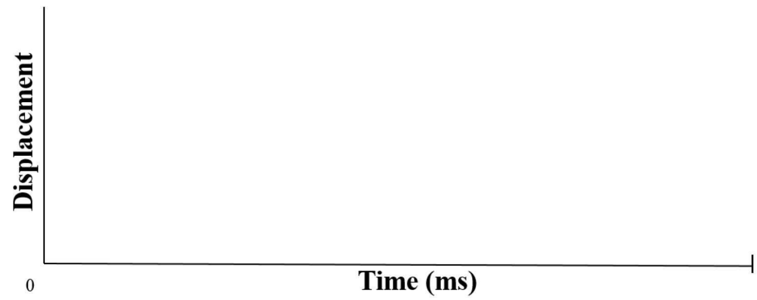
Calculations from EOD and the vehicle manufacturer indicate that one IED recovered can produce a force of $100\frac{kg*cm}{ms^2}$ that has a duration of $20ms$. For the blast seat, m=$100kg$, $\gamma = 20kg/ms$, and $k = 9kg/ms^2$. Assume the blast occurs at $T = 30$.

2.1 Write your model for the displacement of the soldier assuming the given information about the IED.

2.2 Solve your model for $x(t)$ using a Laplace Transform.

2.3 Check your answer by solving your model in *Mathematica*. Provide a sketch of the solution for $x(t)$ from $0 \leq t \leq 100$ ms on the plot below. You may use the *Mathematica* commands provided here.

```
ans=DSolve[{m*x''[t]+γ*x'[t]+k*x[t]== 100*(UnitStep[t-30]-UnitStep[t-50]),
x[0]==0, x'[0]==0}, x, t];
xfinal[t_]=x[t]/.First[ans];]
```



3 Interpreting the Results

3.1 Differentiate your solution for $x(t)$ twice to get an acceleration function. Don't try to write the answer by hand!

a[t_]=xfinal″[t];

3.2 Write out the calculation for the HIC using the formula from the read ahead using the $a(t)$ from problem 3.1 and $T = 30$ and normalize the units for $a(t)$ to "gravitys" (note that $\frac{m}{s^2} = \frac{100\,cm}{1000^2\,ms^2}$ so we can normalize $a(t)$ to gravitys by the multiplication $\frac{1000^2}{100*9.8}a(t)$). This allows the acceleration to take on the common phrasing of "a certain number of g's".

3.3 To compute this integral, you have to use a numerical approximation. We learned how to do this by computing a Taylor Series of a function and integrating term-by-term. Luckily for us, *Mathematica* has a command with a similar capability. Use the following *Mathematica* command to compute the HIC.

$$\boldsymbol{HIC}= \frac{15}{1000}*\left(\frac{1}{15}*\mathbf{NIntegrate[Abs[}\frac{1000^2}{100*9.8}\mathbf{a[t]],\{t,30,45\}]}\right)^{2.5}$$

3.4 Is it likely that the IED described will result in head trauma? Explain.

4 Model Improvement

What are some ways to improve your model? You can discuss improvements even if you don't know the exact way of including them mathematically in your model.