STUDENT VERSION
COMBATING THE WEST AFRICAN EBOLA EPIDEMIC

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STATEMENT

Background

This is background information and a request for a plan of action based on actual US Army involvement in the Ebola outbreak in Africa and as such it is written in the call for present action, not past review.

According to the Centers for Disease Control and Prevention (CDC), the current Ebola virus epidemic has now claimed the lives of 4,912 people, from a total number of 10,114 Infected people. The disease is rapidly increasing its reach within West Africa as evidenced by the fact that 2,101 of the casualties have died during the 31 day period between 23 September and 24 October 2014.

Your unit is currently designated as the Regionally Aligned Brigade for Africa. The brigade S1 (personnel officer) has assigned you as one of the liaison officers to AFRICOM so that you can gain some familiarity with current issues in the AFRICOM Area of Responsibility (AOR) prior to the brigade beginning a series of small unit deployments to Africa. After arriving at the AFRICOM headquarters, you were immediately sent to the still nascent Joint Force Command Headquarters in Liberia, which is responsible for Operation United Assistance. The overall mission of the headquarters is to coordinate US and international efforts at combating the Ebola virus outbreak. You begin working in the J-35 (Operations and Plans) staff section of the headquarters.

Over the next few months you expect to assist the staff in understanding the effects of the current ebola crisis on countries in West Africa and to evaluate some options that the command can implement by coordinating with both the US State Department and local governments. After successfully helping the United Assistance staff recommend a course of action to the government of the Ivory Coast for dealing with a steady increase in the population of Abidjan, you have been

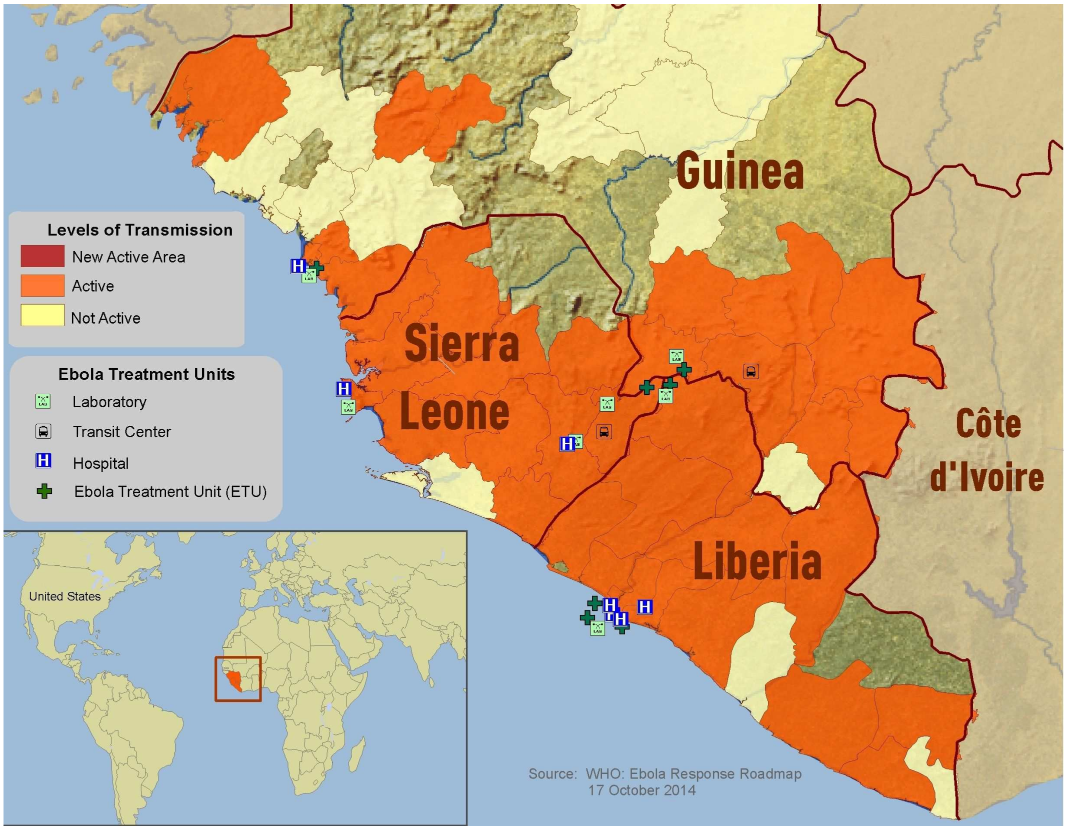

Figure 1. CDC Map of the Ebola Outbreak (OCT 14)

asked to turn your efforts to Liberia. As of 24 October, the country of Liberia has the highest number of reported cases (4,665) and deaths (2,705) where both of these numbers are viewed as under-reported by medical authorities in West Africa. The Liberian government is now seeking assistance in determining the best courses of action to combat the spread of Ebola in the country, and the President of Liberia, Ellen Johnson Sirleaf, is requesting additional aid from the United States.

General Model

A widely used model in epidemiology is the compartmental model, where people are stratified into different states based on their health at a certain time. One powerful compartmental model is the SEIR Model which has four compartments: people who are **Susceptible** to disease, **Exposed** and incubating the disease, **Infected** and contagious with the disease, and **Recovered** from the disease. Each compartment is modeled by a differential equation that describes the rate of change of people in that compartment as a function of the number of people in the other compartments. These four linked differential equations combine to make a system of differential equations that can accurately describe the spread of the Ebola virus within a population. Specifically:

$S(t) \equiv$ The proportion of the population Susceptible to the Ebola virus at time t

$E(t) \equiv$ The proportion of the population Exposed and incubating the Ebola virus at time t

$I(t) \equiv$ The proportion of the population Infected and contagious with the Ebola virus at time t

$R(t) \equiv$ The proportion of the population Recovered from the Ebola virus at time t

$t \equiv$ Time, measured in days

$m \equiv$ Money, in millions of dollars, allocated to different counter-Ebola programs

$\beta \equiv$ The transmission rate of the Ebola virus between Susceptible and Infected people. β is a function of two variables: $\beta = \tau\bar{c}$

$\tau \equiv$ The probability of transmission after contact between an Infected person and a Susceptible person

$\bar{c} \equiv$ The average daily number of contacts that an Infected person has with Susceptible people

$\sigma \equiv$ The daily proportion of Exposed people who become Infected $\left(\frac{1}{\sigma} = \text{average incubation duration}\right)$

$\nu \equiv$ The daily proportion of Infected people who exit the Infected state $\left(\frac{1}{\nu} = \text{average infection duration}\right)$

$f \equiv$ The fatality rate for the virus

$$\begin{aligned}
\frac{dS(t)}{dt} &= -\beta S(t)I(t) \\
\frac{dE(t)}{dt} &= \beta S(t)I(t) - \sigma E(t) \\
\frac{dI(t)}{dt} &= \sigma E(t) - \nu I(t) \\
\frac{dR(t)}{dt} &= \nu(1-f)I(t)
\end{aligned}$$

In this system, people who are exposed to an Infectious person exit the Susceptible state at a rate β, and enter the Exposed state while the virus incubates. Once the incubation period ends, these individuals exit the Exposed state and enter the Infected state, at a rate σ. People exit the Infected state at a rate ν. Of those people, a proportion of them, $(1-f)$, enter the Recovered state, and a proportion of them, f, die from the disease.

> Pick two of the above equations and be prepared to provide a rationale as to why they successfully (or unsuccessfully) model the phenomena in question.

The transitions for individuals passing between the four health states are relatively simple to describe. However, the dynamics of the model are significantly impacted by the coefficients that drive the transition rates. One field within epidemiology works with health data collection to statistically determine the parameters that most accurately match the current situation. The following set of parameters reasonably describes the dynamics of the Ebola virus outbreak in Liberia, as of November 10, 2014.

$$\tau = .2; \quad \bar{c} = 2.0; \quad \sigma = 0.1; \quad \nu = 0.2; \quad f = 0.70$$
$$S(0) = 0.999; \quad E(0) = 0.0002; \quad I(0) = 0.0002; \quad R(0) = 0.0006$$

The Liberian government is seeking international funding to aid in combating the Ebola epidemic and it can allocate the money towards three different programs. Each program has a different effect on the coefficients in the model. Furthermore, the government can allocate the funds between any combination of the three programs below:

1. Protection and Awareness Program: This program will fund and supply basic protective equipment and awareness campaigns where the government issues protective supplies to more communities for the home health treatment of Ebola victims along with information and awareness about Ebola treatment, transmission facts, and proper burial procedures. This is crucial, since there are not enough treatment centers for all of the victims. This program assists by decreasing the probability of transmission of the virus from an Infected person to a Susceptible person. However, the primary risks with this program are cultural push-back against changes in burial procedures, and the difficulty of implementing safe practices around infected individuals even when equipped with protective equipment.
2. Contact Tracing Program: This program will fund contact tracing teams. This is the process of having investigators go door to door after a confirmed case of the Ebola virus is reported to find out those individuals the victim has been in contact with, and to initiate monitoring and/or quarantines for those individuals. This is a critical step in trying to decrease the total number of possible contacts that an Infected individual has with Susceptible individuals. The risks with this program include the difficulty in recruiting individuals to deliberately track down an Infected person's contacts, the logistical hurdles in accurately following up on contacts in hard to reach places within Liberia, and the possibility that even if an Infected person's contacts are identified, it is difficult to monitor the health of all of those contacts.
3. Treatment Units: This program will fund the construction of more Ebola treatment centers thereby increasing the availability of relatively safe spaces for Exposed and incubating patients in addition to space for individuals who are already Infected. For Infected individuals, these treatment units provide an opportunity to slightly decrease the fatality rate through aggressive treatment. For possibly Exposed individuals, these units provide a way of limiting future contacts by having a controlled environment. Risks for this program include societal worry about the Ebola virus being more contagious in a healthcare environment and the possibility that the treatment units will not be located close enough to future virus flare ups.

In addition to analyzing data about the current outbreak, the CDC and the World Health Organization (WHO) maintain data on how different disease mitigation strategies affect the spread of disease. This historical epidemic data, combined with an assessment of current Liberian disease mitigation efforts and costs, can help the Operation United Assistance staff develop a prediction of how future funding to the three programs above changes the model. In general, the parameters relating to incubation time (σ) and recovery time (ν) are functions of the virus and cannot be changed by mitigation efforts. However, the amount of money each program receives can modify τ, $\bar{c}$, and f.

Funding Program	**Result on the Coefficients**	**Specific Change (m in millions of dollars)**
Protection and Awareness	Decrease τ	$\Delta\tau_{\text{Awareness}}(m) = -0.01(\ln(\frac{m}{5} + e) - 1)$
Contact Tracing	Decrease in $\bar{c}$	$\Delta\bar{c}_{\text{Tracing}}(m) = -0.035(\ln(2m + e) - 1)$
Treatment Units	Decrease in f Decrease in $\bar{c}$	$\Delta f_{\text{Treatment}}(m) = -(\frac{1}{2000})m$ $\Delta\bar{c}_{\text{Treatment}}(m) = -(\frac{1}{20000})m^2$

For example, if the government devoted 10 million dollars to funding contact tracing teams:

$$\Delta\bar{c}_{\text{Tracing}}(10) = -0.035(\ln(2 * 10 + e) - 1) = -0.0743109$$
$$\bar{c}_{\text{NEW}} = \bar{c}_{\text{OLD}} + \Delta\bar{c}_{\text{Tracing}} = 2 + (-0.0743109) = 1.92569$$

So, the impact of that decision would be a new value for average daily contacts which means that each Infected person will encounter fewer Susceptible people, which will slow the spread of the Ebola virus.

Part I - Equilibrium and Stability Analysis

Before you begin a numerical analysis of the system of differential equations, it is useful to consider a stability analysis of the long-term behavior of the system.

(a) Find the equilibrium points of the system of differential equations.

(b) Find the Jacobian for the system of differential equations.

(c) Evaluate the stability of each equilibrium point in the system.

(d) Discuss the results you obtained in (a)-(c) and the behavior of the system.

> Due: Submit answers to the different components of Part I as Appendix A to your report for Part II. Submit any supporting electronic work.

Part II - Numerical Analysis

The Liberian government, like others in the region, recently received a 50 million dollar grant from USAID for use in immediately combating the spread of the Ebola virus. Members of the government are seeking advice from the Operation United Assistance staff on analyzing the best use of the funds. The Liberian government hopes to make significant improvements in combating the virus during the next few months, and consequently wants to focus this USAID grant on minimizing the number of fatalities during the next 5 months. Your unit, the Regionally Aligned Brigade, will assist with implementing the selected course of action (COA), and after your assistance in evaluating fund use in the Ivory Coast last month, you have been asked to help the staff evaluate possible allocations.

A course of action consists of allocating x dollars towards the first program, y dollars towards the second program and z dollars towards the third program. The sum of the allocated funds must be less than or equal to 50 million dollars.

Using your model, evaluate possible uses of the funds and develop courses of action for the Liberian government on how they should spend the USAID grant of 50 million dollars. You must develop one course of action per group member. For example, if your group has three members, develop three COAs. You will present your courses of action to the Operation United Assistance operations officer, J3, who will review your courses of action before submission to the Liberian government. You must mathematically justify your selections for courses of action, address why you did not include other options, and you must be prepared to brief it to someone not well-versed in the mathematics of a system of differential equations. Consider how you will compare choices, present your data, create visualizations, and concisely explain the math.

> Due: Submit a concise, neatly organized 1 page paper 12 point font, single spaced, and documented in accordance with the MLA style that summarizes your presentation. Include as appendices any supporting work and relevant explanations. Prepare a 15 minute briefing for the J3 outlining your methodology and proposed courses of action.

Part III - Sensitivity Analysis

The J3 is concerned about the fact that data on the spread of Ebola in Liberia is not abundant, and the fact that the given coefficients for November 10, 2014 might not be exact. Specifically, the J3 is worried that if the CDC's estimates for τ, $\bar{c}$, and f are not correct, then the selected course of action might not turn out as expected. These three parameters each have a margin of error of three percent. Select one of the courses of action that you developed in Part II (each group member must select a different COA), and conduct an uncertainty analysis to determine if there are any significant impacts from an error in the estimations of τ, $\bar{c}$, and f. For example, from the description above in the Model section, if you allocated 10 million dollars towards the Contact Tracing program you would find a new value for $\bar{c}$ and would find a 3 percent range about this value in which to consider the results.

$$\bar{c}_{\text{NEW}} = 2 + (-0.0743109) = 1.92569$$
$$[1.92569 - .03 * 1.92569] \leq \bar{c}_{\text{NEW}} \leq [1.92569 + .03 * 1.92569]$$

So, you need to re-evaluate your model to consider a value of $\bar{c}$ that can be anywhere in this interval, along with similar changes in τ and f based on your course of action. Discuss whether this uncertainty would make you re-think your course of action development.

> DUE: Submit a SEPARATE concise, neatly organized 1 page paper, 12 point font, single spaced, and documented in accordance with the MLA style that guides the reader through the processes you have completed for Part III. Submit all supporting electronic work.