www.simiode.org
SIMIODE Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

STUDENT VERSION
ROCKS IN THE HEAD

Brian Winkel
Director SIMIODE
Cornwall NY USA

STATEMENT

Sensitivity is a subject that invites notions of change, e.g., how sensitive is a system to a slight change in a variable? How discerning are we to change around us? If we could change the wattage to a light bulb some 5 feet from us from 100 to 102 watts, would we notice the change? How about from 100 watts to 500 watts?

We offer a description of a project which measures the ability of individuals to sense the discernible difference between the masses of small rocks. Basically, we lay out a number of small rocks. It is best if these rock samples are from the same "geological stock," i.e. have the same density and color, and that they be smooth, for sharp edges have the effect of appearing heavier as the edge cuts into the bare hand holding the rock used to assess its relative mass, thus distorting one's perception (mass discernment) with another (pressure).

This experiment will require a number of volunteers to each conduct an experiment. The experiment consists of comparing the mass of a number of small rocks to a 100 g mass and simply saying, for each rock, if it has more mass or less mass than the 100 g mass and reporting observations on a score sheet, one sheet per individual. We shall summarize the data for our rock collection. Suppose we have n rocks and compute the percent of our population which says rock i, $i = 1, 2, \ldots, n$ has more mass than the 100 g mass. A discerning population should have a low percentage report that low mass rocks, say 20 g, have more mass than 100 g mass. Similarly, a high percentage might be expected to say a 180 g rock has more mass more than a 100 g mass.

Equipment needed

- 5-10 small rocks or stones (use stones of the same density or a "geological stock," try to get them smooth without jagged edges, and make them in the 50-150 g range);

- 100 g mass from an old science lab scale set (use this as a standard mass or in lieu of the lab piece you can use a rock of known mass as the standard with which you compare the other rocks);
- shoebox or small container to hold the rocks and the standard mass;
- pencils, attached by string to the table or box, and forms (described below) which are used to anonymously collect data;
- table on which to set out this equipment (table should be in a public area where students, faculty, and staff can see it, but in a protected area where it would not be subject to vandalism); and
- ballot box used to collect respondents' forms.

The experiment

The goal of this project is to determine how good people are at distinguishing between different masses. Accordingly, set out some 5-10 rocks in the 50-150g range and a standard mass of 100g. Label the rocks with a marker in random order, 1, 2, 3, . . . , 9, say. Do not label them in increasing mass order! Identify the standard mass and its actual mass for the respondents. We offer data from an experiment we conducted over 35 years ago with 9 rocks.

Ask participants to indicate whether or not the sample rocks are of greater mass than the standard mass. Provide a slip of paper in which to ask them some information you might want for later references, e.g. their major, their position (student-undergraduate or graduate), faculty and discipline, administrative, staff, etc. Do not ask for names unless you wish to have a contest for best guesses.

The real information is respondent opinion on whether the mass of the rocks, numbers 1, 2, 3, . . . , 9, are more than the standard mass. Provide a form such as the following:

Sample rock number	1	2	3	4	5	6	7	8	9
Check if mass $>$ 100 g									

Set the collection table out for whatever time is reasonable to get sample data. One week will be sufficient, especially if there is a sign, with words like 'Rock-A-Rama', 'Let's Rock', 'Science Experiment', or 'Which Major is More Perceptive', and some catchy graphics.

Once the data are collected, compute the percentage of respondents who say that a given rock has mass greater than the 100 g standard mass. Plot this percentage against the actual mass of the given rock for each rock.

Statement of the problem and first actions

We will attempt to fit a curve (or curves) through the response data, perhaps attempting to differentiate groups with a parameter or feature in the graphs we obtain for different groups. We might even wish to rank the groups based on this criteria. For example, can geology majors differentiate rock mass better than political science majors?

Once the data are collected, compute the percentage of respondents who say that a given rock has mass greater than the 100 g standard mass. Then plot this percentage against the actual mass of the given rock. If all respondents had perfect knowledge, e.g., a scale, the plot of the returned data would look like Figure 1.

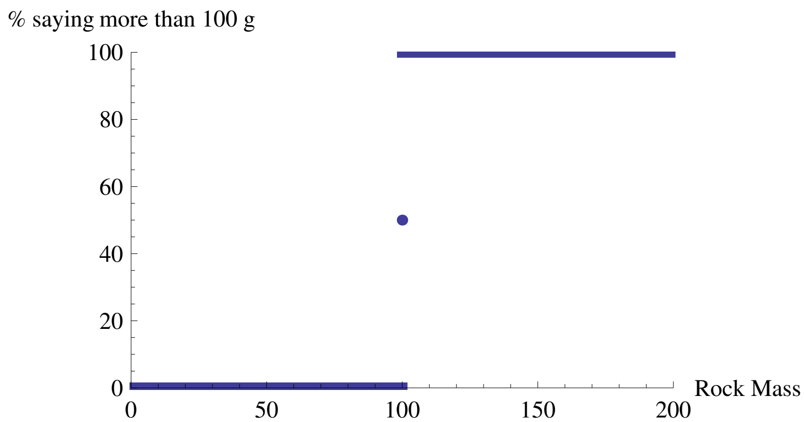

Figure 1. Plot of responses for a group that got it 100% right all the time for whatever rock masses come there way.

1) Think about it: can you explain the dot at the point $(100, 50)$ in Figure 1?

If all respondents guessed then the plot of the returned data could look something like Figure 2.

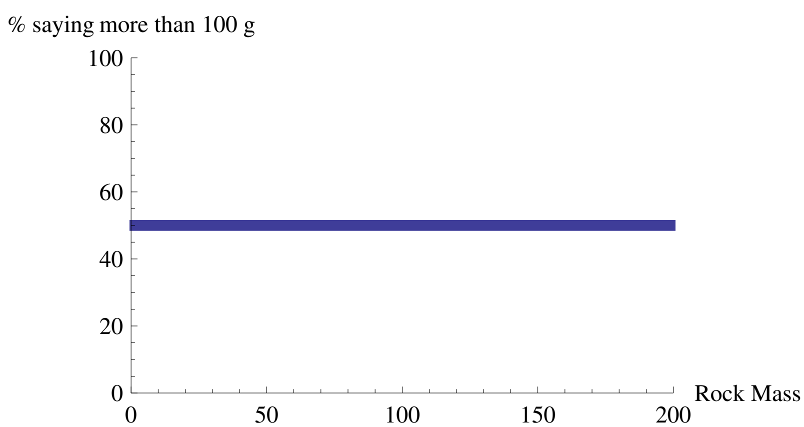

Figure 2. Plot of responses for a group that just guesses all the time.

In reality, neither perfect (Figure 1) nor no knowledge (Figure 2) exists in a population. Rather, as shown in Figure 3, we see a plot of responses for a reasonable (not perfect) population. Notice the sigmoidal shape going from low values for percentage (saying low mass rocks have more mass than 100 g) while there will be a sharp rise around 100 g rocks and then a leveling off as we approach higher and higher mass rocks to about 100% of the respondents saying a HUGE mass rock has more mass than 100 g.

A possible model for this might be the sigmoidal shaped solution to the logistic equation (1) where $M(s)$ is the percentage of respondents who say a rock of mass s has more mass than 100 g mass. Notice that $M(0) \sim 0$.

$$M'(s) = rM(s)(100 - M(s)), \qquad M(0) = M_0\,. \tag{1}$$

2) Explain why $M(0) \sim 0$ is probably true.
3) Explain why the differential equation (1) is reasonable. Consideration of several issues might help you in your explanation.
   a) What happens around the mass of 100 g?
   b) What happens as the mass gets larger and larger? smaller and smaller?
   c) How big can the percent who guess a large rock's mass correctly actually go?

In (2) we see the solution to differential equation (1). You should be able to solve this by hand with separation of variables and partial fraction techniques or Mathematica's `DSolve` command.

4) Solve (1) now and compare your answer to (2).

$$M(s) = \frac{100 M_0 e^{100rs}}{M_0 e^{100rs} - M_0 + 100} \tag{2}$$

Rock Mass (s in g)	% saying Rock Mass > 100 g mass ($M(s)$)
10	2
40	10
70	21
90	45
100	50
120	65
125	75
150	90
175	98

Table 1. Summary of data collected from students at Albion College over 40 years ago.

Table 1 shows some data students collected over 35 years ago from fellow students at Albion College (a small liberal arts college in Michigan). If you cannot collect your own data then you can use this data. Incidentally, the students determined that the geology students performed the worst!

Further, we offer a plot of the data from Table 1 in Figure 3 where the sigmoidal shape of the logistic equation can be seen. Figure 4 renders a plot of how good your model might just be in predicting the performance of these students.

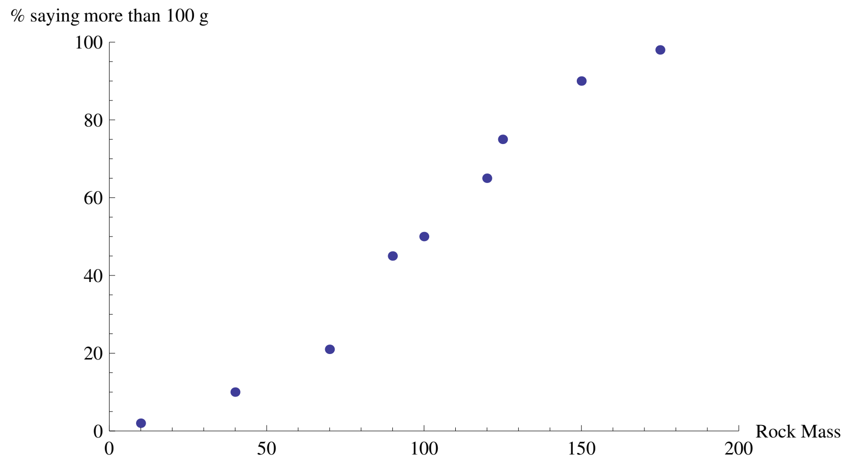

Figure 3. Plot of the data from Table 1 in which % of respondents who said the rock of given mass is more than 100 g is plotted against the mass of that rock.

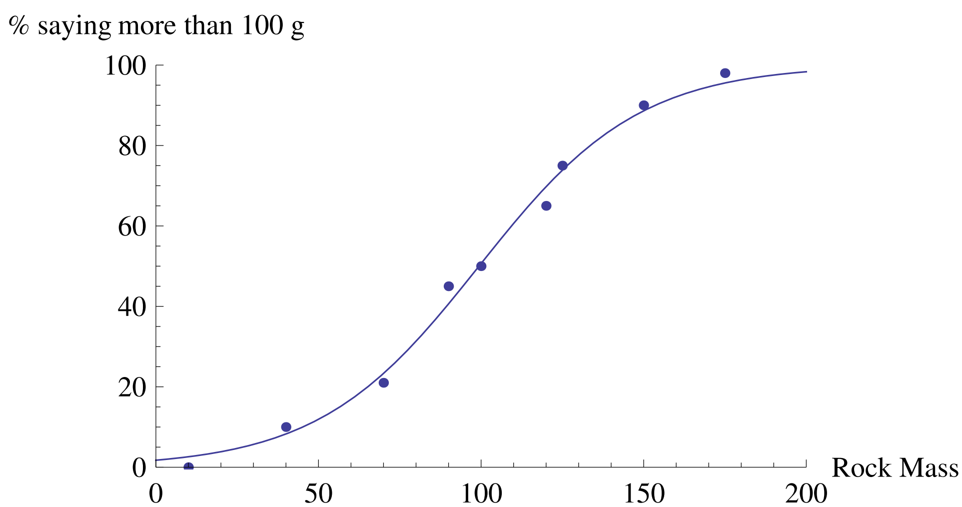

Figure 4. Plot of the data from Table 1 in which % of respondents who said the rock of given mass is more than 100 g is plotted against the mass of that rock and then compared to a good model on the same axes.

5) Determine the parameters r and M_0 which give rise to the best model fit for the data in Table 1 using the logistic equation (2). When compared to the actual data your model should look something like the comparison offered in Figure 4.

6) For the data offered in Table 1 modify it (we do not recommend doing this with real data - it could get you in trouble!) (i) so that we have a very poor group of respondents - worse than our given group, and then (ii) so that we have a very good group of respondents - better than our given group.

7) Now think hard. What makes them better? Give at least three criteria you might use to measure good sensitivity from a group in this experiment and apply one to show why your offerings are in line with your criteria.
8) Go ahead and set up this experiment (we give some practical considerations below) and ask your fellow students who pass by their major and obtain their full responses as to whether or not each rock in your set up has more mass than the 100 g brass mass. Then determine a model for each set of majors and determine which major is best. Remember you design the criterion. Be sure to compare your best major and see if they are really best in all your criterion for best.

Practical considerations in setting up a rocks experiment

1. Use rocks from the same type of rock, if possible. The density of the rocks should all be the same so that appearance is not a part of the decision, e.g., big low density rock offers different sensory possibilities than a large high density rock.
2. Use smooth rocks, for jagged edges "dig into" the skin and effect perception, as the sensation of the jagged edge on nerves in the hand trigger a response saying, "This rock is really heavy."
3. If you are going to set up the collection box for several days take it in at the close of business for janitorial services might remove it.
4. Empty the collection box several times each day.