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SIMIODE Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

STUDENT VERSION
EVAPORATION - ISOPROPYL ALCOHOL AND WATER

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STATEMENT

Evaporation is the conversion of liquid into vapor or gas at the surface interface of the liquid and surrounding gaseous environment. The rate of evaporation of a liquid is the amount of volume lost from the liquid due to evaporation per unit time over a fixed patch of surface area that of liquid. We are going to help you model evaporation of 91% isopropyl alcohol solution with water and verify the model using the data from 6 scenarios in which the liquid sits in a flat Petri dish with constant circular cross sectional area. We then consider one scenario in which the liquid is in a cone and hence has a changing cross sectional area as evaporation takes place. We will then ask you to model the evaporation of plain water in a Petri dish to determine the rate of evaporation of water from which we can use all this information to determine the rate of evaporation of pure isopropyl alcohol.

A MODEL

Consider evaporation. It is a complicated process and many variables are involved, e.g., temperature, humidity, fluid which is evaporating, air pressure. Each of these may contribute to the specific rate, but we would like to develop a direct or simple model for evaporation and we suggest that knowing just how fast the volume changes as liquid evaporates would be a worthy consideration. Thus if $V(t)$ is the volume of our liquid, say in a flat Petri dish of constant circular cross sectional area, then upon what (geometrically, for we could presume all the other variables are held constant) would this rate of evaporation, $V'(t)$, of the liquid depend? rate of

Activity (1)

We offer several plausible models for the rate of evaporation, i.e. the change in volume, and ask you to offer one of your own that is plausible and one that is highly implausible. In each case, for our considered three models, and your own two models, discuss the pros and cons of each.

1.
$$\frac{dV(t)}{dt} = -kV(t)\,, \quad V(0) = V_0\,;$$

2.
$$\frac{dV(t)}{dt} = -kV(t)^2\,, \quad V(0) = V_0\,;$$

3.
$$\frac{dV(t)}{dt} = -kA(t)\,, \quad V(0) = V_0\,;$$

where $V(t)$ is the volume at time t and $A(t)$ is the cross sectional surface area exposed to the air in the container.

Container Description	Mass (g) of Container	Diameter (cm) of Container
Small Petri Dish	18.62	4.80
Medium Petri Dish	39.16	8.90
Large Petri Dish	83.43	13.80
Small Petri Dish Cover	17.98	5.75
Medium Petri Dish Cover	48.15	9.70
Large Petri Dish Cover	85.91	14.70

Table 1. List of containers used to collect data on evaporation of 91% isopropyl alcohol with water solution. All were Petri dishes or Petri dish covers, the latter being of slightly bigger diameter with less mass.

THE DATA

We collected data on evaporation of 91% isopropyl alcohol solution with water in several different containers. We did this in our home office in Cornwall NY USA. Data was collected through the Ohaus Scout Pro 400 g scale using the Scout Pro USB connection Interface kit offered by Vernier Software and Technology's Logger Pro 3 software. In all cases involving the isopropyl alcohol the data was collected and automatically uploaded to a file. This was then copied to EXCEL and converted to Mathematica format for use in analysis. Usually we tried for 30 minutes of data at 4 sample points per minute. The room temperature was in the 68-72^o F range. However, some collection periods went longer and some shorter. None of these variations in the length of observation should have any effect on our parameter estimation or our modeling. We present descriptions of the devices used in Table 1.

The conical funnel used in the collection of data on evaporation of 91% isopropyl alcohol solution and water had an angle of 60^o, formed by the oblique lateral edge and the horizontal when the funnel was stood upright. The diameter across the top of the funnel was 9.8 cm. See Figure 1.

Since the mass density of pure isopropyl alcohol is 0.786 g/cm^3 and that of water is 1 g/cm^3 then a 91% isopropyl alcohol solution has mass density of $0.91 * 0.786 + 0.09 * 1 = 0.80526\,\text{g/cm}^3$.

Time (min)	Mass (g)
0.00	8.52
0.25	8.51
0.50	8.50
0.75	8.50
1.00	8.49
1.25	8.49
1.50	8.48
1.75	8.46
2.00	8.46

Table 2. A portion of the data for the Medium size Petri dish as recorded in the EXCEL spreadsheet 1-26-S-Evaporation-StudentVersion.xls. This needs conversion as is done in the Mathematica notebook, 1-26-S-Evaporation-StudentVersion.nb, to time in seconds and not mass, but volume in cm^3. Be sure to compute the correct mass density in the conversion.

Activity (2)

Pick one of the six data sets (perhaps check to be sure all data sets are "covered" by the class) from the Small Petri Dish, Medium Petri Dish, Large Petri Dish, Small Petri Dish Cover, Medium Petri Dish Cover, and Large Petri Dish Cover collections and perform an analysis using your model of evaporation. Determine all parameters in your model, being sure you know your units, and validate the model as a predictor against the data, perhaps even another data set.

It is reasonable to assume that for comparable models the rate of evaporation should be the same, in units and in magnitude. Check with classmates to see what model they used and what their parameter for rate of evaporation was in their modeling and calculating efforts.

Activity (3)

Use the data on evaporation of water in a Large Petri Dish to perform an analysis using your model of evaporation. Determine the rate of evaporation of water in your model, being sure you know your units, and validate the model.

Now use the rate of evaporation of water from this activity and an estimate of the rate of evaporation of 91% isopropyl alcohol and water solution obtained in Activity (2) to determine the rate of evaporation of pure isopropyl alcohol.

Figure 1. Shown is the apparatus used for collecting data on evaporation when the container was a conical funnel. The bottom of the funnel is sealed and it is supported by cardboard cutout resting on a beaker lip. The apparatus with the Petri dishes was similar, with the Petri dish in place of the beaker and its contents.

Activity (4)

Use the data from the circular conical funnel and build a mathematical model to determine a rate of evaporation here as well. Determine all parameters in your model, being sure you know your units, and validate the model as a predictor against the data.