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SIMIODE Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

STUDENT VERSION
UP DOWN PROJECTILE PASSING

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STATEMENT

Shoot a projectile straight up into the air. Determine the maximum height, H, the given projectile will go. Then consider the time $T(a)$ $(0 < a < 1)$ it takes between when the projectile passes distance $a \cdot H$ going up and then coming down. Develop $T(a)$ as a function of a. Finally, see if there are any relationships involving these two times at which the projectile passes through the same distance, going up and coming down.

Articulation

Consider a projectile, projected upwards in a uniform gravitational field with no resistance.

a) Say the initial velocity is 100 m/sec and the gravitational field is due to earth's gravity, i.e. $g = -9.81\text{m/sec}^2$ downward.

   We fire a projectile directly up from ground zero.

   (i) Determine the maximum height, H, the projectile attains.

   (ii) For heights $h = 0, 0.1H, 0.2H, 0.3H, \ldots, 0.9H, H$ determine the time (from $t = 0$) it takes to reach height h (on the UPward path) and the time (from $t = 0$) it takes to reach height h again (on the DOWNward path). Remember, what goes up must come down!

   (iii) Look at the numbers you get and make a conjecture about the relationship between these times and the height h.

b) Test your conjecture with another initial velocity. Do you get the same results? the same relationship? Does the initial velocity seem to enter your relationship?

c) Verify your conjecture you made in (a) (iii) for the general equation for a projectile fired vertically with constant acceleration a, initial velocity v_0, and initial vertical displacement x_0.