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STUDENT VERSION

MODELING THE SMOKING PROCESS OF SOUTHERN BARBECUE

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STATEMENT

Introduction

Barbecue is a staple in Southern cuisine, and there are many types of meats as well as preparation and cooking styles among barbecue aficionados. In most Southern states, pork is the meat of choice while Texans prefer beef. Whether pork or beef, the cuts of meat used in Southern barbecue are typically high in fat and connective tissues. As such, these cuts require great care to prepare and thus are usually the most inexpensive of all cuts of meat.

The two primary cuts of pork used in Southern barbecue are ribs and front shoulders. Being the smaller of the two, ribs are generally desired because of their preparation time compared to shoulders. Pork ribs used in Southern barbecue include both spare ribs and loin back (or "baby back") ribs. Spare ribs are generally larger and have more fat and connective tissue than loin back ribs which makes them desirable for most barbecue pitmasters.

Pork shoulders can be separated into two categories, namely the picnic ham and the Boston butt. Boston butts are typically the more desirable cuts from the shoulder since they have more marbled fat (which keeps the meat moist and flavorful). Boston butts are generally between 5-10 pounds each, and can take between 6-24 hours to cook depending on the target temperatures of both the meat and cooking device. Boston butts are the typical cuts used to make "pulled pork" barbecue.

The primary cut of beef used in Southern barbecue is brisket. The brisket is a cut from the lower chest of the cow and is extremely tough and fatty due to the fact that it supports about 60% of the

body weight of standing and moving cattle. Untrimmed beef briskets typically weigh between 8-20 pounds (depending on the size of the cow), and they generally take the longest time to prepare and cook of the meats used in Southern barbecue.

Whether they are cooking pork ribs, Boston butts, or beef briskets, barbecue pitmasters usually have at least one common strategy – "low and slow". That is, in order to make these fatty and tough cuts of meat desirable, the fat and connective tissues must be rendered away. This is most easily done by cooking the meat slowly at a low temperature (usually between 225° and 250° degrees Fahrenheit). This "low and slow" process transforms fatty and tough cuts of meat into moist and tender culinary masterpieces.

Ribs, Boston butts, and beef briskets can all be cooked in conventional ovens and slow cookers. However, authentic Southern barbecue requires one ingredient that these devices cannot provide – smoke. True Southern barbecue infuses smoke into the meat while it is being cooked, and this is most easily done using barbecue "pits". The styles, sizes, and prices of barbecue pits are almost endless, but all pits have the common feature of providing smoke to the meat while it cooks. The smoke provided by these pits comes from burning wood, and there are varieties of hardwoods used by barbecue pitmasters which include, but are not limited to, hickory, pecan, oak, mesquite, apple, cherry, and peach. Each of these woods has a distinct flavor, and the types of wood used are typically determined by preference and availability.

Data Collection

In order to mathematically model the smoking process of large cuts of meat like Boston butts and briskets, we use the "Stoker" by Rock's BBQ [1]. Rock's BBQ generously donated this device to the author in 2008 for the purpose of scientific data collection, and Figure 1 illustrates the original (wired) version of the device.

Figure 1. Rock's BBQ Stoker device with attached thermometers.

The Stoker was designed to connect a blower to the device in order to stoke the fire when the temperature falls below a minimum value. It has an easy to use web interface, and it is also accessible using the Telnet protocol over Transmission Control Protocol (TCP). Using the Telnet protocol, the temperature of both the smoke chamber and the meat can be monitored (virtually) continuously with temperatures updated about every five seconds. The author has written Perl software to access

the temperature data over Telnet, record the data into a data file, and serve the raw and entire session graph of the data in real-time.

The cut of meat used in the collection process for this paper was a 10.7 pound beef brisket. Figures 2 through 5 illustrates the preparation, smoking, and final slicing of the brisket.

Figure 2 shows the untrimmed brisket immediately out of the packaging. A beef brisket has two components – a leaner part called the "flat" and a fattier part called the "point". Figure 3 shows the brisket after it has been trimmed and separated. Figure 4 shows both the beef brisket as well as a Boston butt after being placed in the smoker. Figures 5 and 6 illustrate the finished sliced barbecue brisket.

Assignment

The provided data file [2] contains three columns of tabular data. The first column is time t in hours, the second column is the smoke chamber temperature $h(t)$ in degrees Fahrenheit, and the third column is the internal meat temperature $f(t)$ in degrees Fahrenheit. Use the provided data file to

1. Graph $f(t)$ and $h(t)$ versus time t.
2. Identify the interval where the rate of change of $f(t)$ is small (i.e., the interval where the temperature of the meat stalls). This partitions the time duration into three intervals $[t_0, t_1)$, $[t_1, t_2)$, and $[t_2, t_3)$ called Stage I, Stage II, and Stage III, respectively where $t_0 = 0$ and $t_3 = 11.35$.
3. Explain why the following logistic model is a reasonable model for $f(t)$ on Stage I and Stage III.

$$\frac{dy}{dt} = \lambda(y - k)(K - y)$$

4. Verify that

$$y(t) = k + \frac{K - k}{1 + De^{-\lambda(K-k)(t-\gamma)}}$$

is the general solution to the following differential equation.

$$\frac{dy}{dt} = \lambda(y - k_1)(K - y)$$

5. Estimate the parameters k_1, K_1, D_1, λ_1, and γ_1 for which

$$g_1(t) = k_1 + \frac{K_1 - k_1}{1 + D_1 e^{-\lambda_1(K_1-k_1)(t-\gamma_1)}}$$

is a reasonable model for $f(t)$ on Stage I.

6. Estimate the parameters k_3, K_3, D_3, λ_3, and γ_3 for which

$$g_3(t) = k_3 + \frac{K_3 - k_3}{1 + D_3 e^{-\lambda_3(K_3-k_3)(t-\gamma_3)}}$$

is a reasonable model for $f(t)$ on Stage III.

Figure 2. Untrimmed Brisket.

Figure 3. Trimmed/Separated Brisket.

Figure 4. Brisket and Shoulder.

Figure 5. Sliced Brisket.

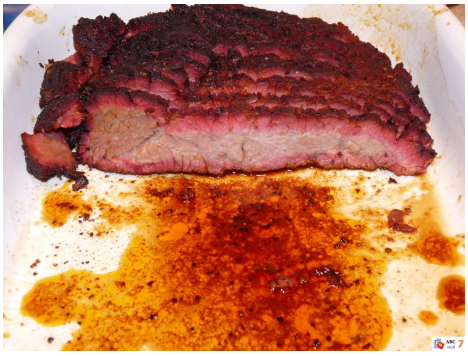

Figure 6. Finished Brisket.

7. Find a linear function $g_2(t) = m_2t + b_2$ defined on Stage II such that

$$g(t) = \begin{cases} g_1(t), & t_0 \le t < t_1 \\ g_2(t), & t_1 \le t < t_2 \\ g_3(t), & t_2 \le t < t_3 \end{cases}$$

   is continuous on $[0, 11.35)$.
8. Graph $f(t)$ and $g(t)$ versus t.
9. Determine the root mean square error between $f(t)$ and $g(t)$ on $[0, 11.35)$.
10. Discuss whether or not you would expect the internal temperature of another 10.7 lb beef brisket to behave exactly like $f(t)$.

Conclusion

Smoking Southern barbecue is a time consuming process, and when executed correctly, transforms fatty and tough cuts of meat into moist and tender masterpieces. Not only is the finished product delectable, but the cooking process can be modelled mathematically to confirm and explain the existence of a stall (or plateau). There are several theories as to the cause of meat stalling when cooked slowly, and the stall induces a natural delineation of three stages throughout the cooking process – namely pre-stall, stall, and post-stall. This study has demonstrated that the pre-stall and post-stall stages can be modelled by logistic differential equations while the stall itself can be modelled linearly.

REFERENCES

[1] Rock's BBQ.2016. Rock's BBQ in Fremont, CA. `http://www.rocksbarbecue.com`.

[2] The data file, bbq.d, is available in the electronic version of this Modeling Scenario.