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STUDENT VERSION
M&M - DEATH AND IMMIGRATION MYSTERY

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Abstract: We describe a classroom activity in which students use M&M candies to simulate death and immigration. Each student conducts an experiment with an immigration rate unique to that student - of that student's choice. Collected data on generation or iteration and population is then passed to another student and the receiving student builds a mathematical model, usually a linear first order, difference or differential equation, and estimates the death and immigration parameters.

STATEMENT

Consider a regular size bag of M&M candies, not peanut, just regular. Usually there are about 55 pieces in each bag. They are of different colors, but each piece almost always has an "m" pressed on one side and not the other. Hence, there are distinctions between the sides. Let us conduct an experiment (a death and immigration simulation) on the M&M's.

Death and Immigration Mystery Model

You will conduct this simulation in which M&Ms either live or die, while at each iteration a certain number (between 6 and 16) of M&M's (your choice of number, call that number MI) immigrate into the population. After opening the bag of candy your teacher will tell you to place an original number, $M(0)$, of M&M's pieces into a small cup, call this cup the Active cup, to start the simulation. You should have a second Reserve cup for reserve M&M's and a paper plate for capturing the M&M's as they are tossed. Use the sheets provided for recording your data - one copy for you and one copy to pass along to another student.

The Experiment: Gently shake the M&Ms out onto the paper plate to catch the M&Ms. We determine for each M&M if it lives or dies. If the m shows on top this M&M goes "belly up" and dies, otherwise there is life for this M&M. Upon death you should remove the dead M&M's from the population (set these aside into the Reserve cup as we may need

them). THEN you select a number of M&M's from the Reserve cup to immigrate back into the population, call that number MI. **Use this same number of immigrants in each generation you conduct**. Your immigration number should be between 6 and 16. Add these immigrants to the surviving M&M's in the plate. Now count and note down the number of M&M's who survive or have immigrated as the number of M&M's at the start of the next generation in Table 1 or on sheets provided to you. Then put these M&Ms back into your Active cup for the next iteration. Do this over and over for, say, 10 generations or iterations.

a) Before starting the experiment describe what you think will happen. Care to make a prediction?

b) State your assumptions about the physical activity. Offer up some assumptions in support of your description and some which might not have that much to do with what you expect to happen. Many times assumptions are run into each other, so break the assumptions down to their simplest form, i.e. do not use words like "and."

c) In Table 1 record what happened and compare this with what you thought would happen. You may also record the iteration or generation and # M&Ms at start of iteration in two columns on both index cards. Label the columns at the top (Generation and # M&M's) as shown in Figure 1 of one such simulation. On the back of the index card, in tiny print, in the lower right, in very small print (we are stressing small print), put the number of immigrants YOU used in each iteration. This number is the mystery to be discovered with modeling by the student to whom you pass along one of your index cards. It will help confirm the model with the data you pass along.

Modeling Death of M&Ms	
Iteration	# M&Ms at start of iteration
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Table 1. Modeling death and immigration of M&Ms.

d) Compare your description/prediction with what actually happened.

e) Indicate which of your assumptions were reasonable and played a role in the experiment.

Model Building and Parameter Estimation

Let us attempt to build a mathematical model of this activity, being sure to include death and immigration aspects. Suppose we were to define the number of M&Ms alive at the start of iteration n as $a(n)$. What values would n take? What would $a(0)$ be in this case? Do you "know" what $a(1)$ would be?

Based on the observations and your assumptions, produce a reasonable formula for $a(n)$, i.e. offer up a discrete function $a(n)$, in the one variable, n, for $n = 0, 1, 2, \ldots$. Incidentally such a model for $a(n)$ could be in *closed form*, something like $a(n) = n^2 - n$, or it could be in the form of a *recursion* equation $a(n) = 3a(n-1) - 7$. Which form is easier to construct for this simulation?

f) Discuss with colleagues around you the reasonableness of your function models for $a(n)$.

g) How will you measure your "success" as a modeler in this situation? Check with others in the class and see if you are on target with your model.

h) Use your model to the estimate immigration rate. You might either presume death rate (what value? and in what manner?) or attempt to estimate that as well.

i) Go ahead and check on just how good your model is at predicting your experiment and determining the mystery immigration rate. Also, re-examine your rationale in (f).

APPENDIX-SAMPLE SIMULATION DATA

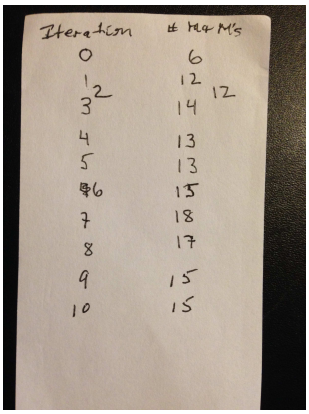

Figure 1. Sample generation and number of M&M's in each generation from death and immigration simulation.

Death and Immigration Mystery Simulation

Material

1 small bag M&M's (regular not nut)	2 small cups (paper cups will do)
1 paper plate	2 copies of this sheet for tallying results

Simulation instructions

You are going to conduct a simulation with M&M's as members of your population. The population will experience death and immigration for a number of generations, say 10 as outlined in the table below, and you will record the population at each generation in table below.

Your teacher will assign you an original number of M&M's, $M(0)$, and you will designate a number of immigrants (number between 6 and 16) to use in each generation, MI, Write that number, MI, in small font at lower right hand corner on the back of this sheet.

You will conduct your simulation and record the same data on both copies of this sheet, one for your records and one to pass along to a colleague for analysis.

Steps of simulation:

1. Open a bag of M&M's and place $M(0)$ (instructor designated initial population) M&M's in one cup (Active cup) and the remaining M&M's in the other cup (Reserve cup).
2. Write in $M(0)$ in table below.
3. Gently toss the M&M's from the Active cup onto the paper plate and remove the M&M's which have an "m" facing up, placing them into the Reserve cup.
4. Add MI immigrants to the plate from Reserve cup and count the M&M's on the plate.
5. Enter that tally in the next generation's column designated # M&M's at start of iteration.
6. Place all M&M's from the plate into the Active cup and go to Step (3) above.

Modeling Death of M&Ms	
Generation/Iteration	# M&Ms at start of iteration
0	$M(0) =$
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	