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SIMIODE Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

STUDENT VERSION
CONDENSATION

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STATEMENT

Consider the random motion of 200 particles in a 50 by 50 square with vertices $(0,0),(0,50)$, $(50,50),(0,50)$ in a plane. At each iteration, $n = 0,1,2,\ldots$ each particle moves 1 unit in one direction only, due north, due east, due south, or due west, and it must move.

Particles are initially randomly distributed on points with integer coordinates, but not on the boundaries, i.e. from within the set $S = \{\,(m,n)|\, m,n \in \{1,2,3,\ldots,49\}\,\}$ and at each iteration each particle moves. The number of condensed particles is tallied at each iteration.

If a particle comes in contact with the west, east, or north wall it bounces off by returning to the position just before the bump. If the particle comes in contact with the south or bottom wall it condenses and stays exactly at the position of contact, thus depleting the number of particles which are still randomly moving in the square. In Figures 2-4 we show a snapshot of several iterations of the particles and the plot of the number of condensed particles at each iteration.

We provide several animations for different runs of the simulation and we offer several data sets from such simulations.

1. Offer up a difference or differential equation which models $y(t)$ the number of condensed particles at iteration t.
2. Using the data set with step size 1 in Table 1 (and shown in Figure 1) or from one of the data sets provided estimate the parameter(s) in your model.
3. Confirm in some way the correctness of your model.

Iteration	0	1000	2000	3000	4000	5000	6000	7000	8000	9000	10000	11000	12000
# Condensed	1	75	103	121	129	136	150	160	170	173	179	182	188

Table 1. Sampled data at increments of 1000 iterations from a simulation of duration 10,000 iterations, with step size 1.

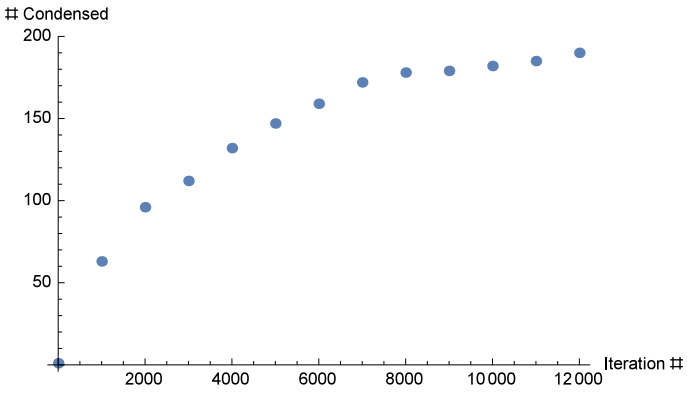

Figure 1. Plot of data in Table 1 of number of condensed particles at iterations, 1000, 2000, 3000, . . . , 11,000, 12000.

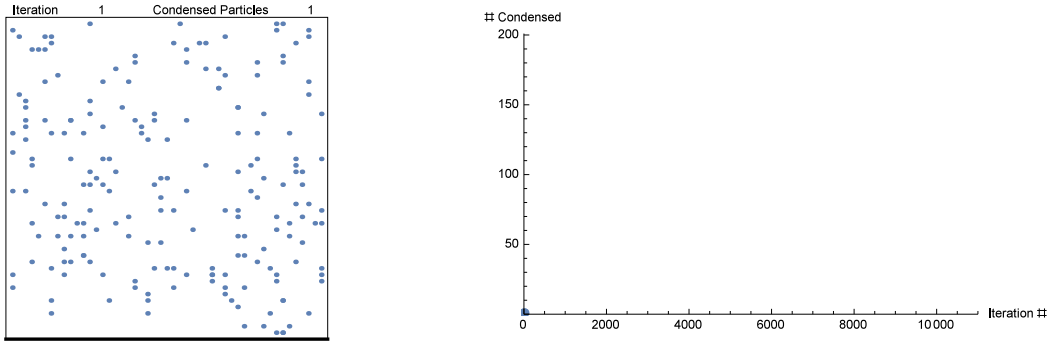

Figure 2. On the left we see the particles in the box at iteration 1 with 1 of the particles condensed on the bottom wall and on the right we see a plot of the accumulated number of condensed particles at given iteration. In our depiction of the particles we do not show accumulating particles which are condensed, rather we presume they are taken away.

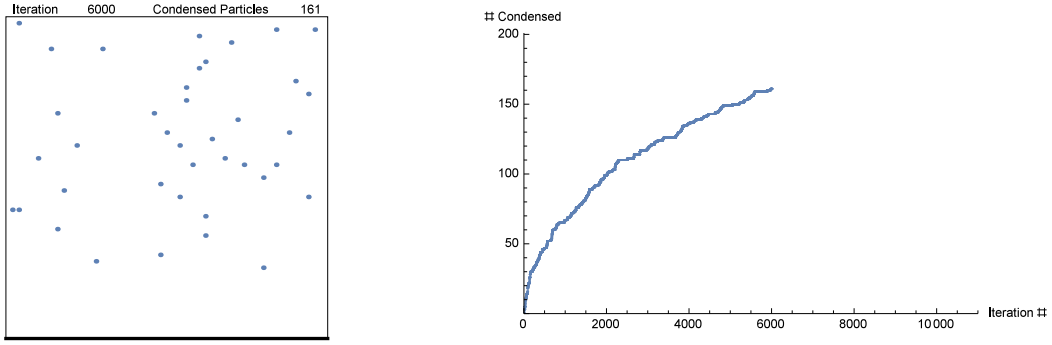

Figure 3. On the left we see the particles in the box at iteration 6000 with 161 of the particles condensed on the bottom wall and on the right we see a plot of the accumulated number of condensed particles at given iteration. In our depiction of the particles we do not show accumulating particles which are condensed, rather we presume they are taken away.

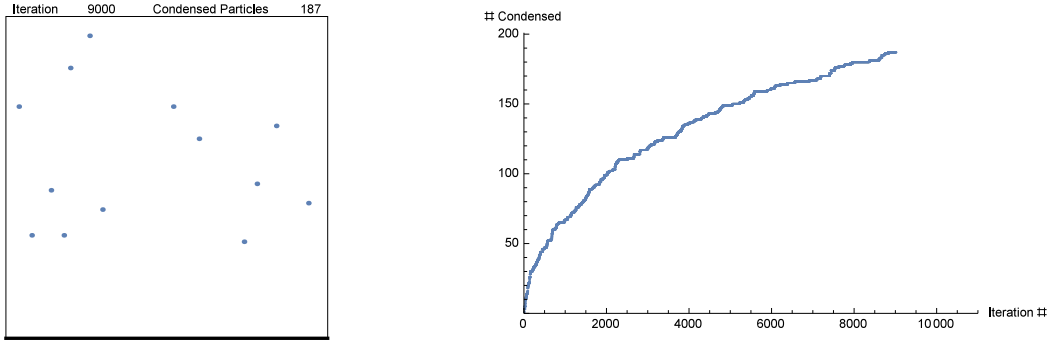

Figure 4. On the left we see the particles in the box at iteration 9000 with 187 of the particles condensed on the bottom wall and on the right we see a plot of the accumulated number of condensed particles at given iteration. In our depiction of the particles we do not show accumulating particles which are condensed, rather we presume they are taken away.