STATEMENT

A baseball player (outfielder) throws a ball with initial velocity 135 ft/s (about 92 mi/hr - less than a good pitcher’s high speed fastball of 100 mi/hr) toward home plate some 400 feet away. Another player (an infielder who cuts the ball off - catches it - from the outfielder’s throw) places himself in direct line between the outfielder and home plate and catches the ball $x$ feet from home plate and then throws (relays) the ball home with initial velocity 110 ft/s.

It takes 2 seconds to accomplish the relay. Assume that due to air resistance the velocity $v = v(t)$ obeys the differential equation (1).

$$v'(t) = -0.1v(t), \quad (1)$$

and ignore vertical motion.

1. Find the time, $T$, it takes for the ball to reach home plate as a function of $x$, the distance from the plate of the infield cut-off player.

2. Find $x$ which minimizes $T$.

3. For a range of infielder capabilities, i.e. initial throwing velocities from 60 ft/s to 140 ft/s, prescribe the cutoff distance $x$ and the total time for the relay to the plate.

4. Consider the infielder who can throw 110 ft/s. Would you rather improve the infielder’s relay time, say down to 1.8 sec, or increase his initial velocity on the ball by 10 ft/s? Explain.