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SIMIODE Systemic Initiative for Modeling
Investigations and Opportunities with Differential Equations

STUDENT VERSION
CIRCULAR ROLLER COASTER

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STATEMENT

Consider a cart of mass m moving through a vertical circle of radius R. We will assume the size of the cart is much smaller than the circle and neglect the rotational inertia of the cart's wheels. The acceleration due to gravity is g (Figure 1).

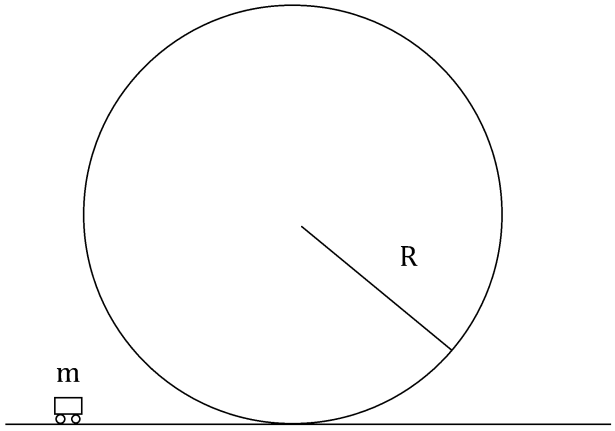

Figure 1. Circular roller coaster.

A typical textbook problem is to calculate the minimum initial velocity of the cart for it to stay on the track throughout the loop [1].

1. Find this minimum initial velocity.
2. Derive the equation of motion for the cart and verify your answer to (1) is correct.
3. Now include the presence of kinetic friction and derive the equation of motion for the cart.
4. Suppose the coefficient of friction between the wheels of the cart and circular track is μ. Verify this equation reduces to the frictionless case (2) when $\mu = 0$ and find the minimum initial velocity of the cart for it to stay on the track throughout the loop in the case of friction.

REFERENCES

[1] Serway, Raymond A., & Jewett, Jr., John W. 2008. *Physics for Scientists and Engineers with Modern Physics, Ninth Edition*. Boston MA: Brooks/Cole.