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SIMIODE Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

STUDENT VERSION
VILLAGE EPIDEMIC

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STATEMENT

In the mid seventeenth century in a small village in England a form of the Plague spread from 3 July through 20 October in one year. We note three classes of individuals: Susceptible, Infective, and Removed. The latter group consists of those who have died from the disease or who developed an immunity from the disease, having already had the disease. We keep track of the following:

$S(t)$ = the number of Susceptibles on day t of the epidemic, note $S(0) = 235$.

$F(t)$ = the number of Infectives on day t of the epidemic, note $F(0) = 14.5$.

$R(t)$ = the number of Removeds on day t of the epidemic, note $R(0) = 0$.

In building a differential equation model we will need to postulate rates of change in each of these populations, i.e. we will need to produce three separate differential equations:

$S'(t) =$ ____________________ ,

$F'(t) =$ ____________________ ,

$R'(t) =$ ____________________ .

We provide data on the epidemic in Table 1.

Let us make two assumptions (you are to defend them) so that we can build such a differential equations model.

- At each instance in time a proportion of the Susceptibles will become Removeds and hence leave the Susceptibles, either by death or immunity.
- At each instance in time a proportion of the contacts between Susceptibles and Infectives will become Infectives and hence leave the Susceptibles.

Time - Days	# Susceptible	# Infectives
0	235	14.5
16	201	22
31	153.5	29
47	121	21
62	108	8
78	97	8
109	83	0

Table 1. Plague epidemic data from mid seventeenth century from a small English village.

1. Now build the three differential equations model for $S'(t)$, $I'(t)$, and $R'(t)$, fully identifying the meaning of each of your terms in the equations and defending their use.

   In each of these differential equations you should have parameters and if you are a good modeler you will find only two parameters.

2. Use the data to estimate the parameters in some fashion and verify the validity of your model.

Incidentally, one could also use difference equations in which one changes the population discretely over time interval, say Δt or $\Delta t = 1$ for daily updates:

$$S(t + \Delta t) = S(t) + \underline{\qquad\qquad\qquad\qquad},$$
$$F(t + \Delta t) = F(t) + \underline{\qquad\qquad\qquad\qquad},$$
$$R(t + \Delta t) = R(t) + \underline{\qquad\qquad\qquad\qquad}.$$

The blanks in the continuous differential equation format and the discrete difference equation format will have the same form, except that the terms in the difference equation format will each be multiplied by Δt as the change in populations will be given for a daily rate and should we elect our change in time, Δt, to be less than 1 for numerical reasons and accuracy, then we need to multiple the daily change rate by Δt.