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STUDENT VERSION
DEFLECTION OF A HORIZONTAL BEAM

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STATEMENT

Students will investigate the vertical deflection of a thin horizontal beam under various end (boundary) conditions. Vertical deflections at points along the beam are measured and it is verified that the deflection curve is a portion of the graph of a polynomial.

Equipment Needed

- One 8 foot piece of $1\frac{1}{2}$ inch by $\frac{1}{4}$ inch plastic moulding (available at Home Depot and other building supply stores).
- A 1 foot by 8 foot strip of paper, with a long line parallel to one long edge and a bit in from it OR with a 1 cm grid printed on it.
- A one foot ruler with a centimeter scale on one side.
- Two rectangular tables in a room with about 3 meters (or 10 feet) of free wall space at the level of the table tops and below.
- Two calculus books OR two other suitably large and heavy books.
- A pencil.
- A computer with Excel on it.

Scenario Number One

Push the two tables against a wall, about 2 meters apart from each other. Lay the strip of moulding across both tables, flat side on the tables, with about 1 foot overlapping onto each table. Place a calculus book on each end to hold it flat against the table. Push the edge of the moulding against the wall. Tape the strip of paper to the wall so that the line (or one grid line) is even with the top of the strip on each end, where it lies on the table.

Remove the calculus books and strip, and move one or both tables so that the distance between them is 1.8 meters and the edge of each table that is toward the other is aligned with a vertical gridline on the paper, if you are using gridlined paper. Lay the moulding strip across the two tables, flat side down and one edge along the wall. Release the ends of the strip so that it sags between the two tables, supported only by the edge of each table on each side. There will be a bit more than a foot of each strip in the air above each table, so it does not exactly model what is called the *pinned-pinned* situation, where the ends are supported, but it is close.

Data Collection

Use the pencil to mark the vertical deflection of the top edge of the strip on the paper, at 10 centimeter horizontal intervals, starting from where the left end of the beam is supported by the table. Try not to change the vertical deflection when doing this. Record in a table the amount of vertical deflection from the line that was at the top edge of the strip at its ends at ten centimeter intervals, from the left end $(x = 0)$ to the right end $(x = 180)$. *Include the vertical deflection of zero at each end.* Make all vertical deflection measurements to the nearest tenth of a centimeter.

We include an Excel spreadsheet, HorizontalBeamData.xls, which includes data collected by the authors from Scenarios One, Two, and Three.

Scenario Number Two

Lay the strip of moulding across just one table, holding the strip flat against the table. This will allow one end to hang free off the table. Adjust the amount hanging off until the free end is 1 meter horizontally from the edge of the table. Place the heavy books on top of the strip to hold it flat against the table, right to the edge. Record vertical deflections at 10 centimeter intervals as before, again being sure to record the vertical deflection of zero at $x = 0$. This is called a *cantilever* or *embedded-free* beam where one end is embedded and one end is free.

Scenario Number Three

Now move one or both tables until they are 2 meters apart from each other. Lay the strip of moulding across both tables, flat side on the tables, with about 1 foot overlapping onto each table. Place a calculus book on each end to hold it flat against the table. Collect and record the data as in the previous scenarios. This is called an *embedded-embedded* beam where both ends are embedded.

Modeling the Data

Let x represent the distance from the left end of the strip, and let y represent the vertical deflection. Create a table of your x and y values for each scenario in Excel. Insert a simple scatter chart of the data for Scenario 1. Right click on any point of the scatter plot to obtain the option of adding a trendline. The data is clearly not linear, so try a polynomial of degree two. Keep trying higher degrees until you find a smallest degree that gives a good fit. Repeat for the other two (end condition) scenarios. What *single* degree seems to give a good fit for all three scenarios?

The Boundary Value Problem

Assuming that the vertical deflection is described by a differential equation, we wish to speculate as to what that equation might be. Assuming also that the solution appears to be a n^{th} degree polynomial, we note that if we took the derivative n times we would obtain

$$\frac{d^n y}{dx^n} = K, \tag{1}$$

where K is some constant. Solving this equation by taking a sequence of anti-derivatives would result in a n^{th} degree polynomial with n arbitrary constants. n additional conditions would then be needed to determine these constants. Let's look at what those conditions are, beginning with the third scenario. We will construct a differential equation with associated (different) boundary conditions in each of the three scenarios, using Scenario Three to estimate our parameters from the data collected in this scenario.

- **Scenario Three** - Each end is held at a vertical deflection of zero, and each end is also horizontal at its very ends. Give these boundary conditions as mathematical statements utilizing the vertical deflection function $y = y(x)$ and any of its relevant derivatives.
- **Scenario One** - Again each end is held at a vertical deflection of zero, but the strip of moulding is clearly not horizontal at its ends. What is (not necessarily obviously) the case is that the strip has no curvature at its ends, which means that the ends are at points of inflection on the fourth degree polynomial curve. What do we know about derivatives at points of inflection?
- **Scenario Two** - The conditions at the table end are the same in this case as in Scenario Three. What about the free end? There is again no curvature at the free end, giving us one condition there. The other condition is beyond the scope of this course, but it is that the *third* derivative of the vertical deflection is zero there.

REFERENCES

[1] Zill, Dennis G. 2005. *A First Course in Differential Equations with Modeling Applications*. Belmont CA: Brooks/Cole - Thompson Learning, Inc.