

**For the 10th Annual Spuyten Duyvil
Undergraduate Mathematics Conference,
Manhattan College**

**Modeling First Teaching of Differential
Equations – SIMIODE**

0930, 16 April 2015 Miguell Hall 207

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Director SIMIODE

www.simiode.org

We present an approach we have used for years and are developing as a web-based community of teachers and learners



Systemic Initiative for Modeling Investigations
and Opportunities with Differential Equations

www.simiode.org

Modeling first differential equations We discuss examples of activities to enable students to develop mathematical modeling skills using differential equations while using technology throughout their learning and doing mathematics.

SIMIODE Team

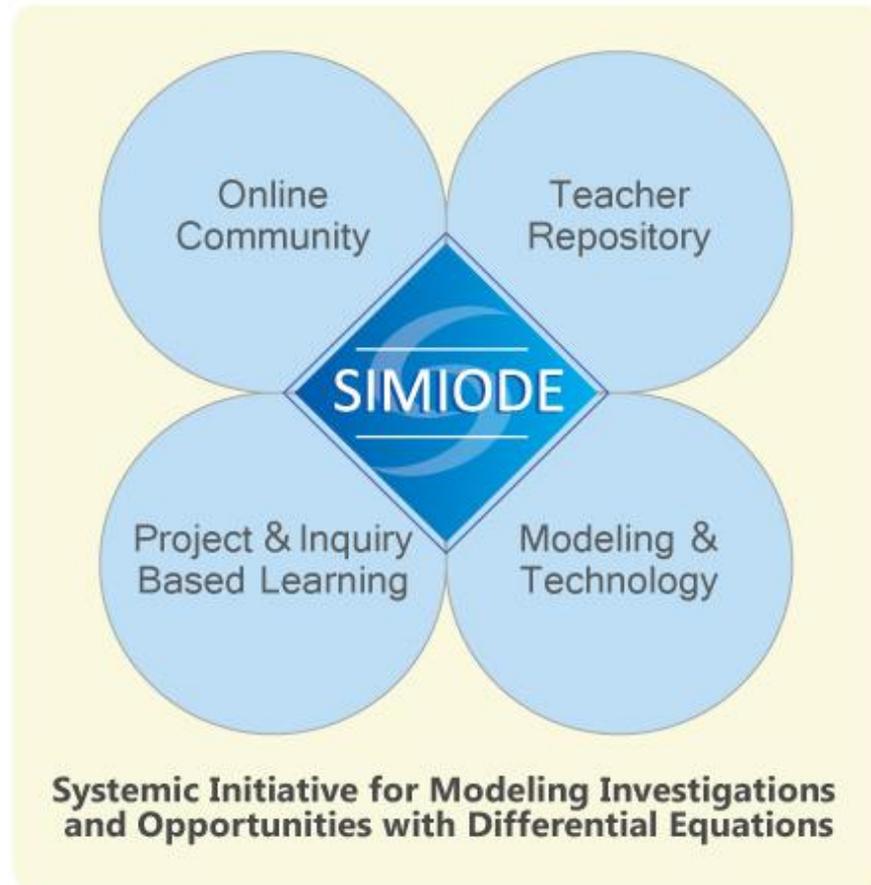
Leigh Noble, Technical Director and applied mathematician

Mark Tourtellott, Technical Director, Art Designer

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Karen Bliss – Quinnipiac U and then VMI, Associate Director

Brian Winkel, Director, Emeritus MathSci, USMA



Eric Sullivan, Carroll College

Ted Wendt, Carroll College

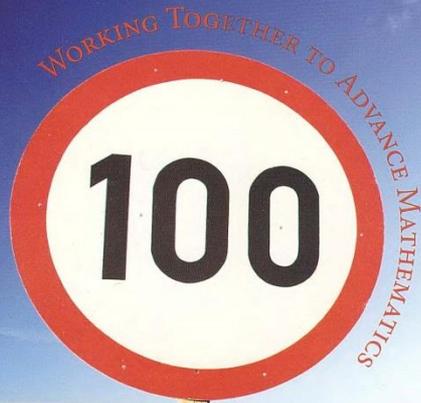
Audrey Malagon, VA Wesleyan College

Board of Contributing Advisors

PREP



Professional Enhancement
Programs of the MAA



MAA 100
MATHEMATICAL ASSOCIATION OF AMERICA
CELEBRATING A CENTURY OF ADVANCING MATHEMATICS

Building Community in SIMIODE—Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

Brian Winkel, Jessica Libertini, Karen Bliss

July 19–25, 2015
Helena, Montana

REGISTRATION FEE: \$325 *per person*
(*\$400 for registrations received after*
June 7, 2015)

SIMIODE is a web-based community, combining the benefits of online-learning communities, teacher repositories, problem-based and inquiry-based learning, and technology into a web-based environment for the teaching and learning of differential equations through mathematical modeling. This workshop is for faculty who want to teach differential equations using the SIMIODE community. During the preparatory component, participants will join the SIMIODE community at www.simiode.org and experience Modeling Scenarios from a student viewpoint, Discussion Activities about important issues, and Teacher Groups for developing materials. During the onsite component, participants will have opportunities to engage as student groups, to realize the potential of SIMIODE, and to share reactions, experiences, and aspirations for engaging students. Colleagues will work in groups to begin the process toward publishing in SIMIODE. After the workshop activities, conversations will continue through SIMIODE's web features (e.g., Discussions, Projects, Newsletter) as PREP participants form their own special group inside the SIMIODE Teachers Group.

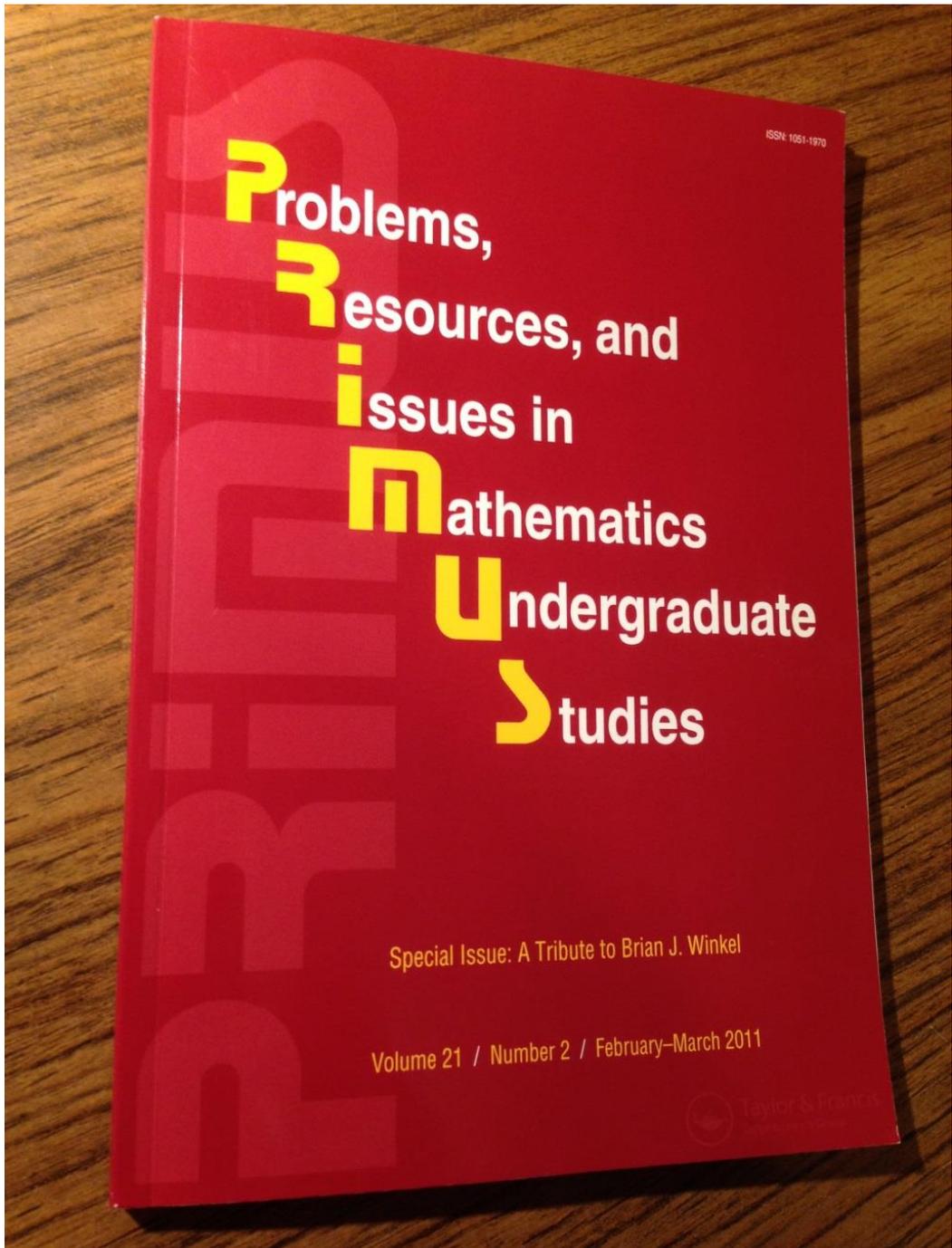
Sources of modeling opportunities with data are all about us.

- (1) Do the Internet thing,
- (2) browse library journals – current and the stacks in fields not known to you,
- (3) talk to colleagues OUTside mathematics,
- (4) do your own, e.g., spread of slime or cookie dough!
- (5) attend other department seminars and conferences,
- (6) see ideas in Teachers Group at www.simiode.org.

Take risks, learn yourself, jump into the modeling process with your students, and have fun learning with them!

Reference: Winkel, B. J. 2013. Browsing Your Way to Better Teaching.

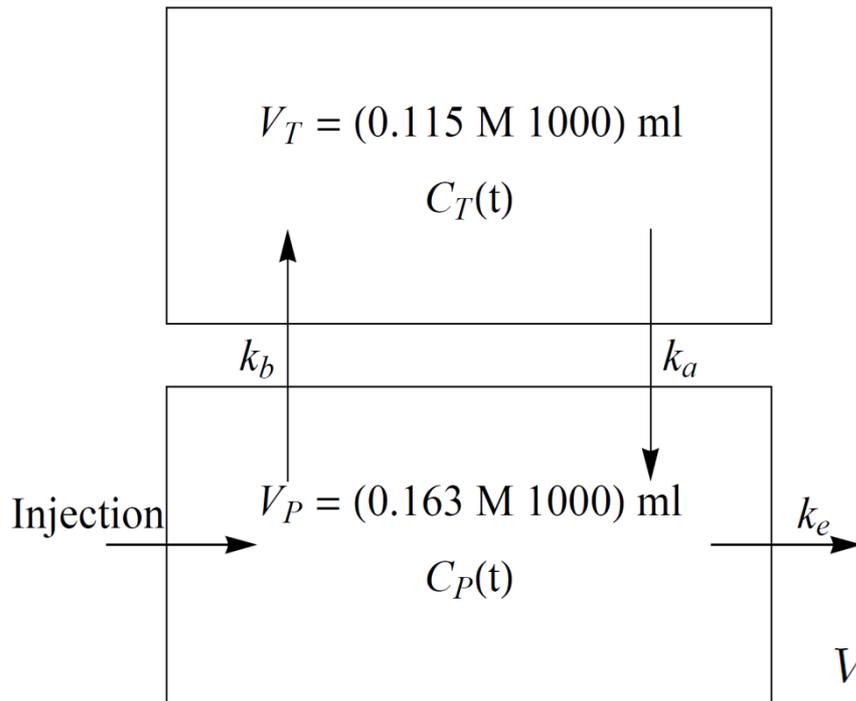
PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies.
23(3): 274-290. Original references in this paper.



A place find resources for teaching

. . . and another place to share your teaching efforts. . .

Modeling Lysergic acid diethylamide (LSD) in the human body



$C_p(0) = 12.2699$, for originally LSD was injected at a concentration of 2,000 ng per kg of body mass for each subject and so we have an initial concentration in the plasma of $12.2699 = 2000M / .163M1000$ ng of LSD/kg of body mass. $C_T(0) = 0$.

COMPARTMENT MODEL

P = Plasma and T = Tissue

$$V_P = \underbrace{(0.163M)}_{\text{kg of plasma}} \cdot \underbrace{(1)}_{\text{liter/kg}} \cdot \underbrace{(1000)}_{\text{ml/liter}} = \underbrace{(163M)}_{\text{ml of plasma}}$$

$$V_P C'_P(t) = k_a V_T C_T(t) - k_b V_P C_P(t) - k_e V_P C_P(t)$$

$$V_T C'_T(t) = k_b V_P C_P(t) - k_a V_T C_T(t).$$

	Time (hr)	0.833	0.25	0.5	1.0	2.0	4.0	8.0
Subject 1	Plasma Conc (ng/ml)	11.1	7.4	6.3	6.9	5.	3.1	0.8
	Perform Score (%)	73	60	35	50	48	73	97
Subject 2	Plasma Conc (ng/ml)	10.6	7.6	7.	4.8	2.8	2.5	2.
	Perform Score (%)	72	55	74	81	79	89	106
Subject 3	Plasma Conc (ng/ml)	8.7	6.7	5.9	4.3	4.4	—	0.3
	Perform Score (%)	60	23	6	0	27	69	81
Subject 4	Plasma Conc (ng/ml)	10.9	8.2	7.9	6.6	5.3	3.8	1.2
	Perform Score (%)	60	20	3	5	3	20	62
Subject 5	Plasma Conc (ng/ml)	6.4	6.3	5.1	4.3	3.4	1.9	0.7
	Perform Score (%)	78	65	27	30	35	43	51

Table 2. Summary of data collected [1, 14] on 5 male volunteers who were administered LSD and then tested on performance (Perform Score (%)) on simple addition questions. Both performance Score and Plasma Concentrations of LSD were recorded at 5, 15, 30, 60, 120, 40, and 480 minutes after the initial infusion of LSD.

This is the formula for $C_T(t)$, the concentration of LSD in the Tissues as a function of time t .

$$\left(\frac{-17.3913 e^{0.5 \left(-1. k_a - 1. k_b - 1. k_e - \sqrt{-4. k_b k_e + (1. k_a + 1. k_b + 1. k_e)^2} \right) t} + 17.3913 e^{0.5 \left(-1. k_a - 1. k_b - 1. k_e + \sqrt{-4. k_b k_e + (1. k_a + 1. k_b + 1. k_e)^2} \right) t}}{\sqrt{-4. k_b k_e + (1. k_a + 1. k_b + 1. k_e)^2}} \right) / k_b$$

Solving the system can be done by hand using eigenvalue approaches or with a computer algebra system (in our case Mathematica) offering a closed form solution.

Take time to notice the forms machines return to us. . . . exponential decay in Tissues.

$$V_P C'_P(t) = k_a V_T C_T(t) - k_b V_P C_P(t) - k_e V_P C_P(t)$$

$$V_T C'_T(t) = k_b V_P C_P(t) - k_a V_T C_T(t).$$

$$SSE(k_a, k_b, k_e) = \sum_{i=1}^i (C_P(t_i) - O_i)^2 \quad (3)$$

Plasma Conc. LSD 25 – ng/ml

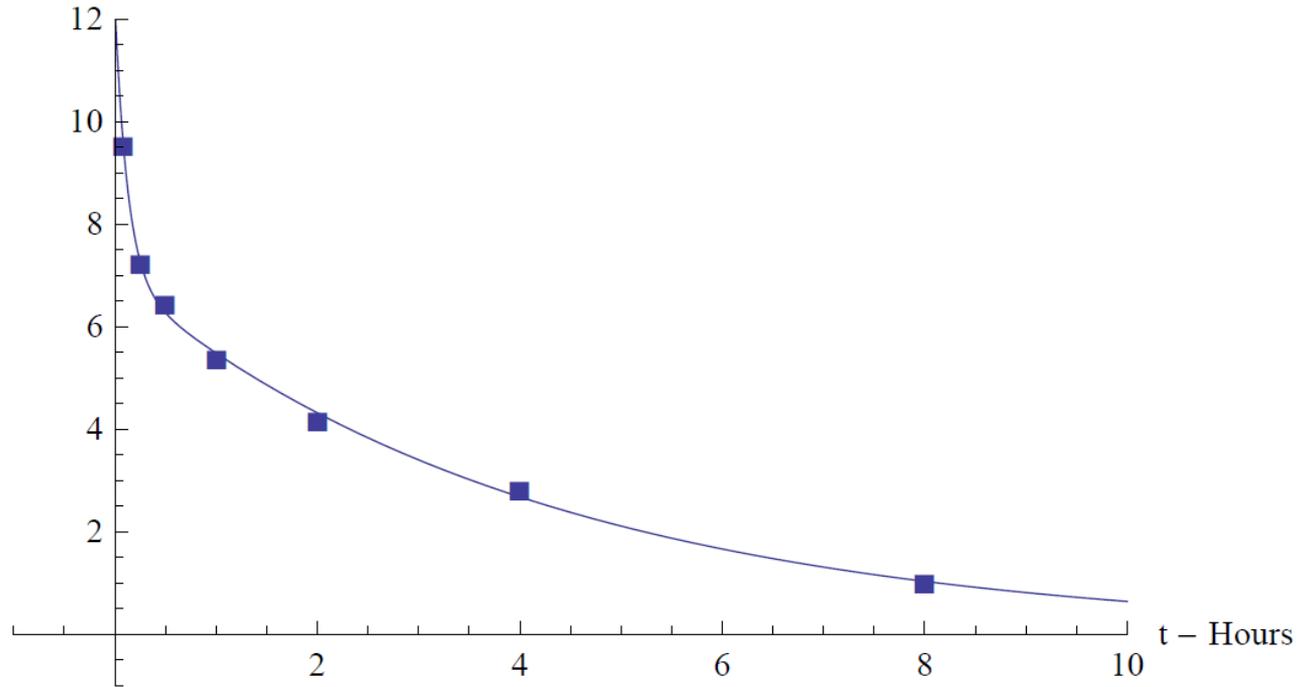


Figure 2. Plot of the observed values of the average concentration of LSD (ng/ml) (squares) and the model built from parameter (k_a , k_b , and k_e) estimates using the solution of the system of differential equations (2) and minimization of the sum of square error function (3).

$$C_T(t) = 0.128905 (55.419e^{-0.238492t} - 55.419e^{-7.99617t}) . \quad (5)$$

Tissue Conc. LSD 25 – ng/ml

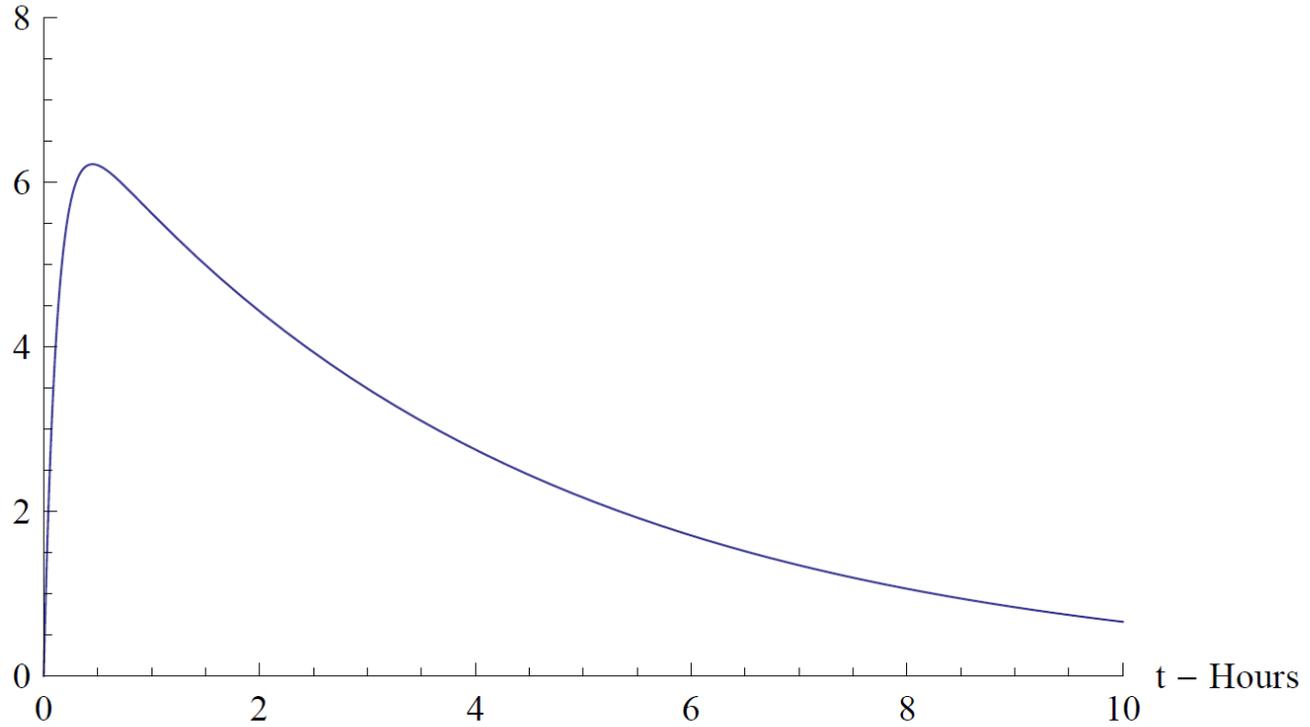


Figure 3. Plot of the model of tissue concentration of LSD in ng/ml. This model is built from parameter (k_a , k_b , and k_e) estimates using the solution of the system of differential equations (2) and minimization of the sum of square error function (3).

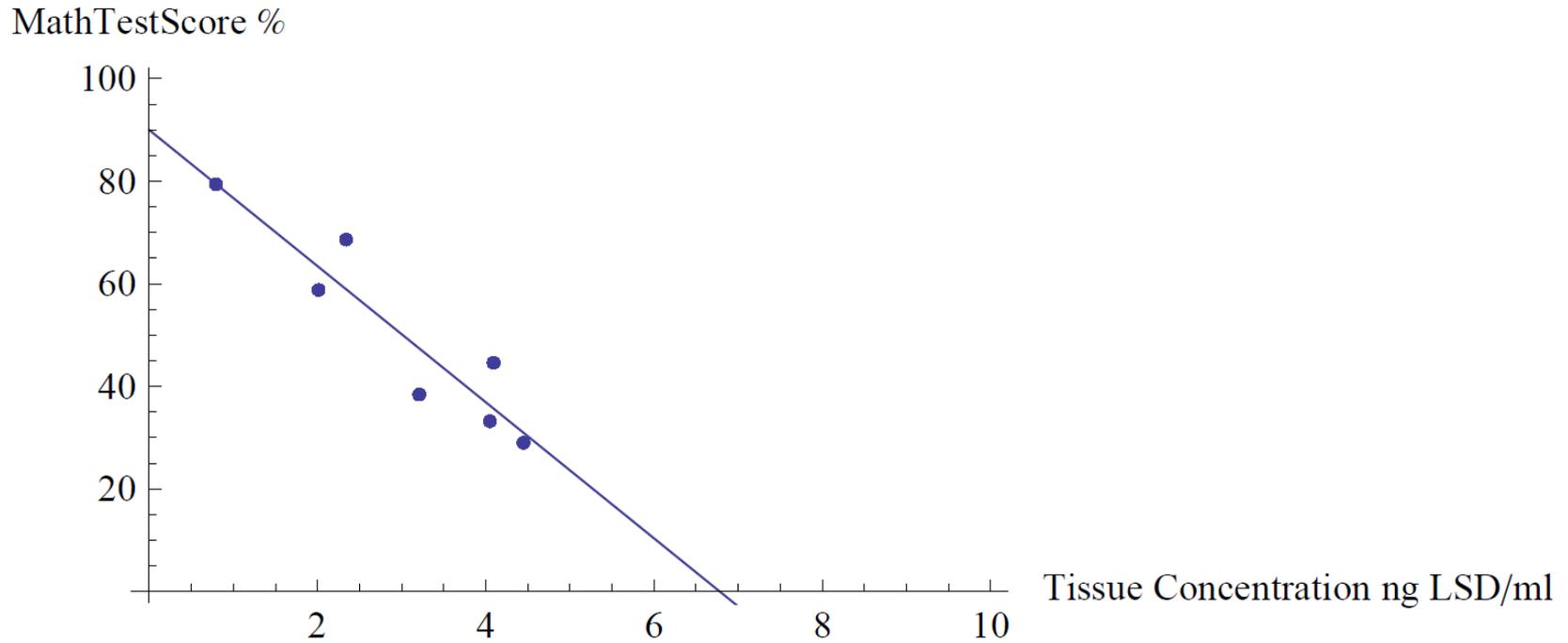


Figure 4. Plot of Performance Score (%) (PS) on the simple arithmetic problems vs. the model prediction of the concentration of LSD in ng/ml in the tissue compartment (CT). Superimposed is the best line of regression whose equation is $PS = 89.1729 - 9.32941CT$. This means for every ng/ml increase in LSD in the tissue compartment the score drops a little over 9 points.

Tuned Mass Dampers

Introduced in MA364 Engineering Mathematics

A Tuned Mass Damper (TMD) is a passive mechanical counterweight for a structure consisting of a moving mass (roughly 1–2% of the structure's mass) which is usually placed in the upper portion of the structure. The purpose of the TMD is to reduce the effects of motion caused by wind or earthquake.

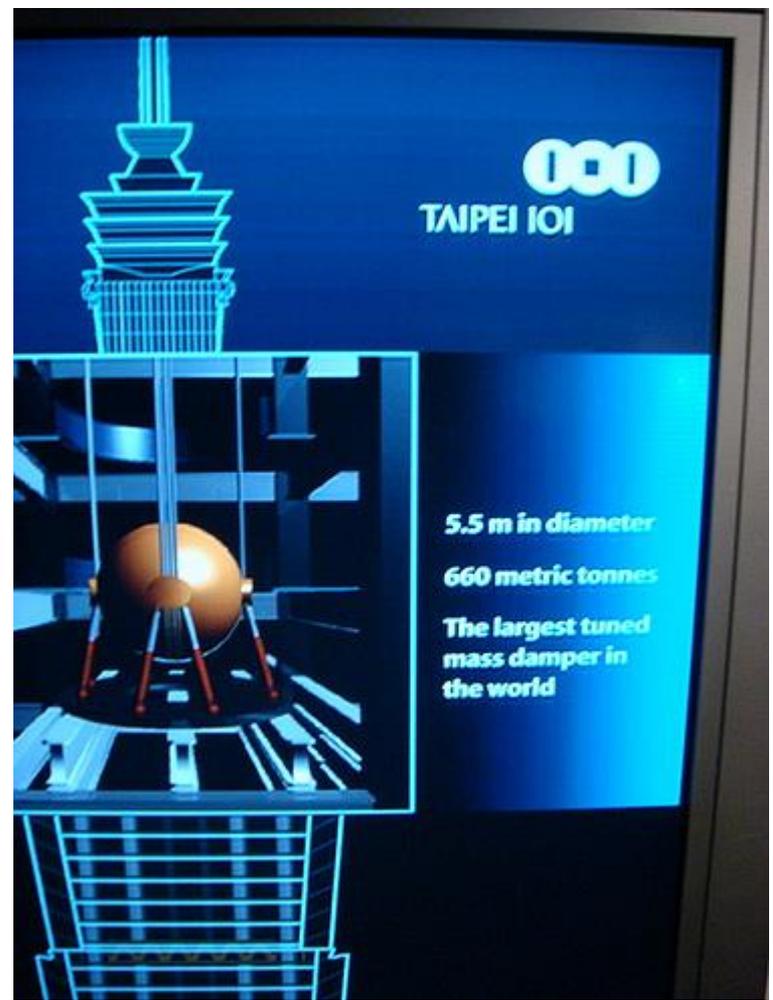
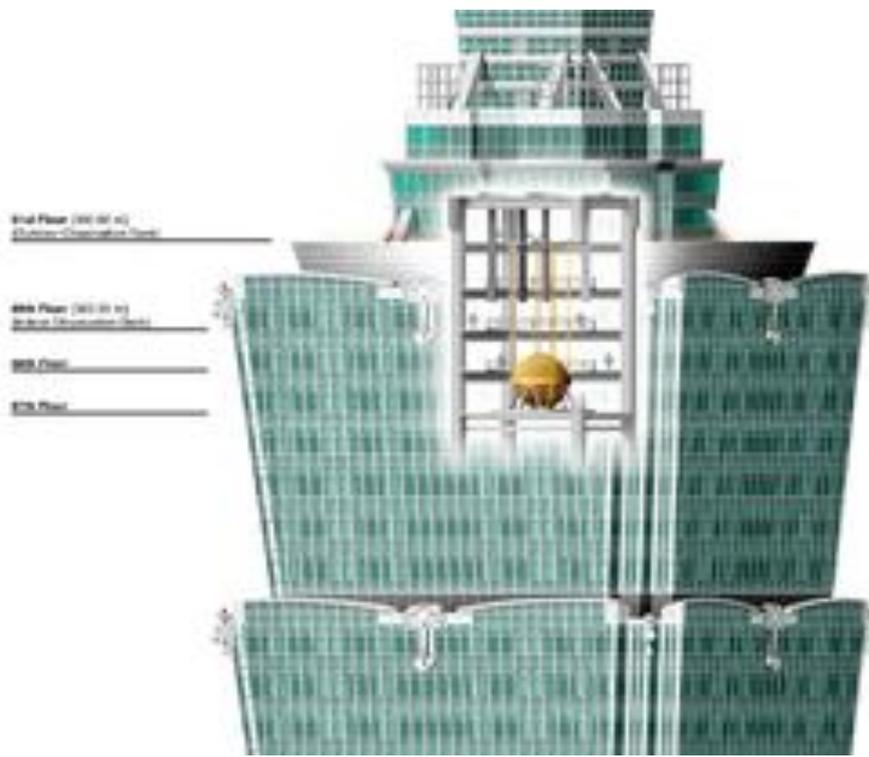
The first uses of TMDs in the United States for large structures was in the John Hancock Building in Boston in 1977 and City Corp Center in New York in 1978.

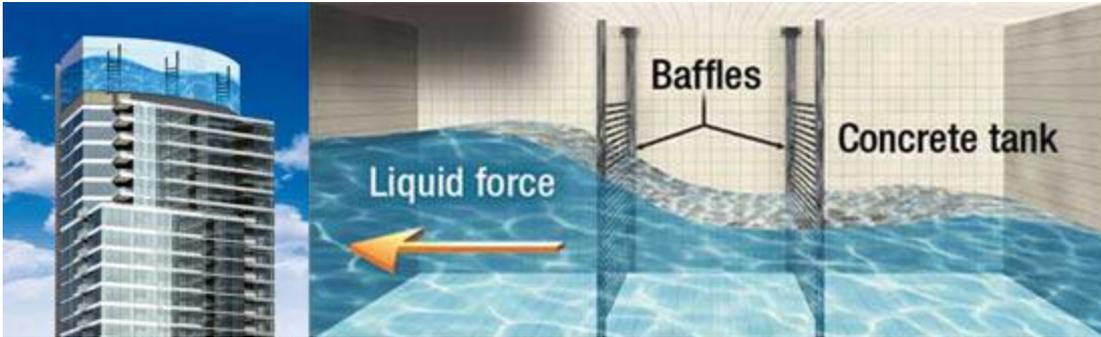
TMD's are used in many, many structures and devices, including buildings and bridges, electric razors, rotating tools, surgery table platforms, etc.

So what is a TMD? Well it is just a system of two second order differential equations And a physical device to implement the results.

TMD's can be in the form of sliding slabs (HUGE) of concrete and steel, sloshing tanks of water, pendula, and more.

Winkel, B. J. and **K. Landry**. 2008. Peak Frequency Responses and Tuned Mass Dampers--Exciting Applications of Systems of Differential Equations. *The UMAP Journal*. 29(1): 15-30.



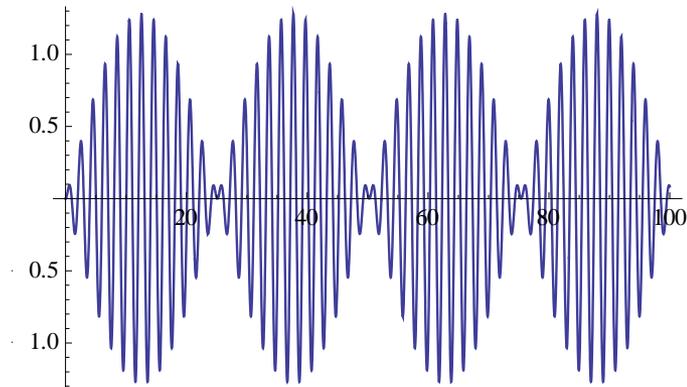


Harmonic Oscillator, e.g., building or bridge OR oboe reed.

$$1 y''(t) + \omega_0^2 y(t) = \cos(\omega t), \quad y(0) = y_0, \quad y'(0) = v_0$$

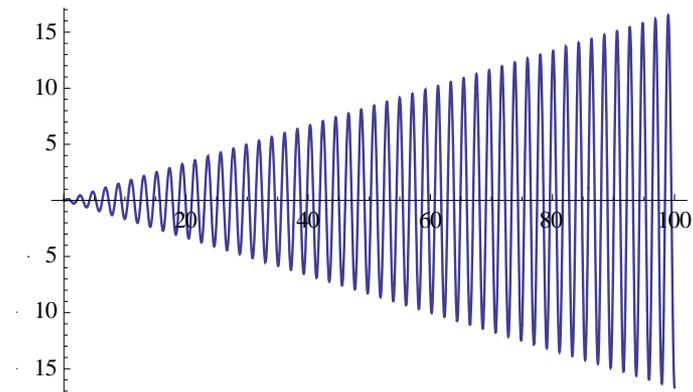
When ω_0 and ω are close we get beats,

e.g. $\omega_0 = 3.0$ and $\omega = 3.25$



When ω_0 and ω are equal we get resonance,

e.g. $\omega_0 = 3.0$ and $\omega = 3.0$



Thus if in 1 $y''(t) + \omega_0^2 y(t) = \cos(\omega t)$, $y(0) = y_0$, $y'(0) = v_0$ we have $\omega_0 = \omega$, i. e. the driver frequency is the same as the natural frequency of the system/structure, then we can have resonance and the system/structure can have giant and dangerous oscillations. For a building this can come from wind or earthquakes. What can we do to mitigate this danger?

Differential equations come to the rescue!

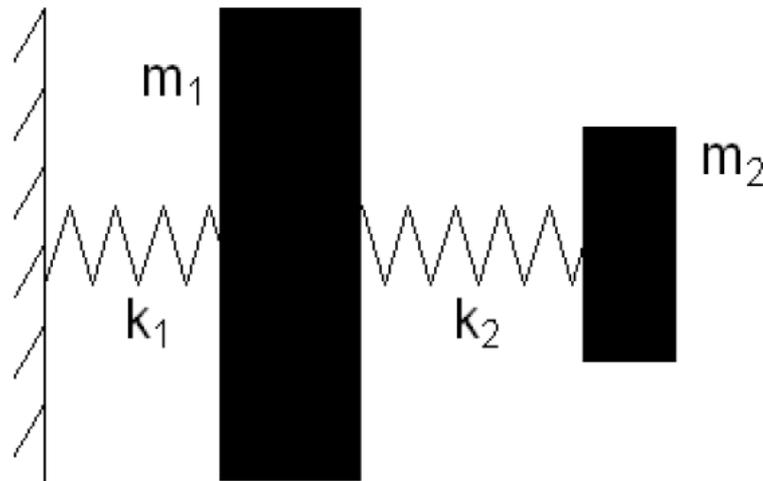
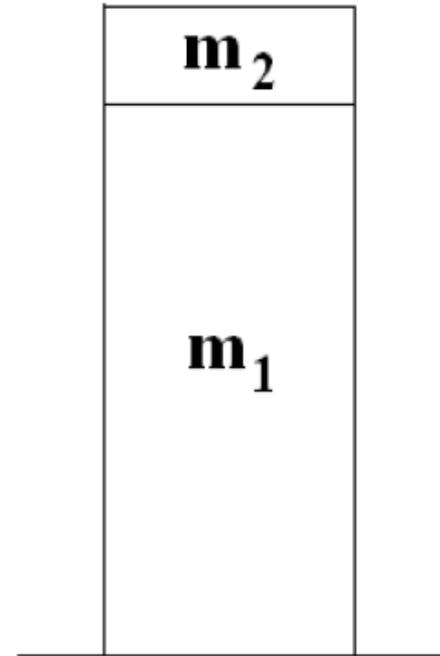
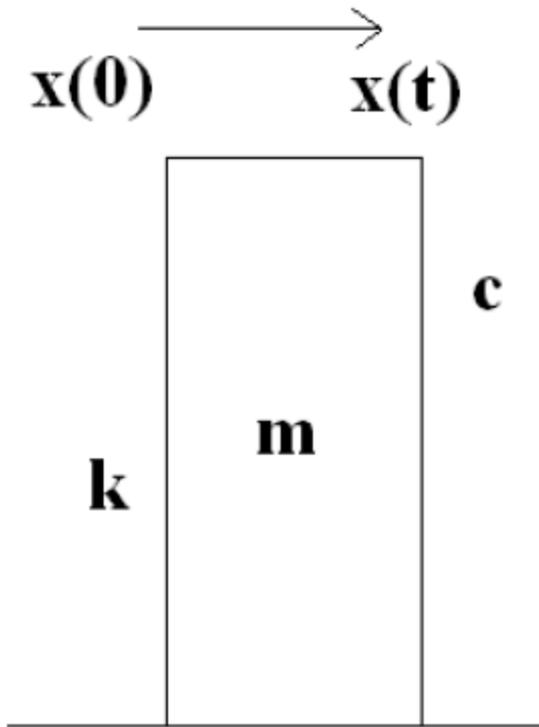


Figure 6: Horizontal depiction of two mass spring system (no damping on either mass) with smaller mass m_2 serving as Tuned Mass Damper.



Depiction of two masses for a structure – smaller mass on top.
K (spring constant) is called stiffness and c is still called damping coefficient.

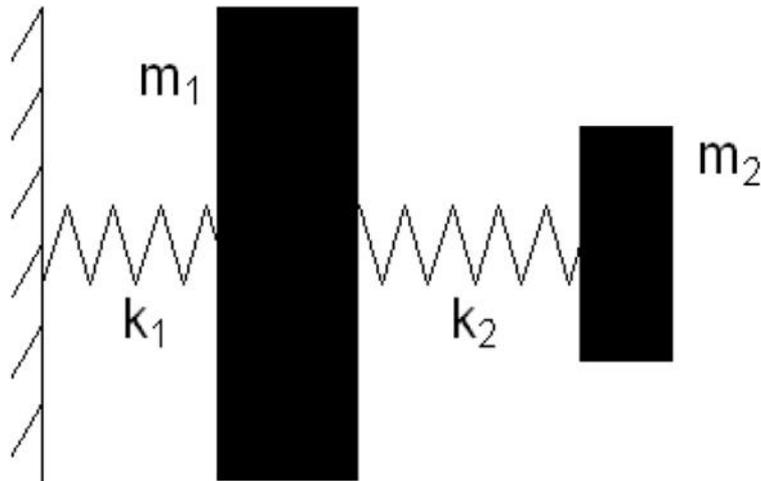


Figure 6: Horizontal depiction of two mass spring system (no damping on either mass) with smaller mass m_2 serving as Tuned Mass Damper.

$$\begin{aligned}
 m_1 x_1''(t) + (k_1 + k_2)x_1 - k_2 x_2 &= F(t) \\
 m_2 x_2''(t) + -k_2 x_1 + k_2 x_2 &= 0.
 \end{aligned}$$

In the simplified case where there is no damping (i.e. $c = 0$) then we might seek to control for resonance by adding a second “tuned” mass.

We now “drive” the larger mass with a force. . wind, earthquake

$$m_1 x_1''(t) + k_1 x_1(t) + k_2 x_1(t) - k_2 x_2(t) = F_0 \cos(\omega t)$$

$$m_2 x_2''(t) - k_2 x_1(t) + k_2 x_2(t) = 0 .$$

If no second mass then we could have resonance ($c=0$) or high amplitude.

So we add a second smaller mass and we “tune” it so that its system has the same frequency as that of the larger mass.

The natural frequency of our structure is $\omega = \sqrt{\frac{k_1}{m_1}}$ and of the added mass the frequency is $\sqrt{\frac{k_2}{m_2}}$. Consider $k_1 = 90$ and $m_1 = 10$ with $k_2 = 0.90$ and $m_2 = 0.10$.

Then both spring systems have the same frequency, namely $\omega = \sqrt{\frac{90}{10}} = \sqrt{\frac{0.9}{0.1}} = 3$.

Now, by examining the solution of our system

$$\text{mass } m_1 : \quad -\cos(t\omega) + \frac{909}{10}(a \cos(t\omega) + b \sin(t\omega)) - \frac{9}{10}(c \cos(t\omega) + d \sin(t\omega)) \\ + 10(-a \cos(t\omega)\omega^2 - b \sin(t\omega)\omega^2) = 0$$

$$\text{mass } m_2 : \quad -\frac{9}{10}(a \cos(t\omega) + b \sin(t\omega)) + \frac{9}{10}(c \cos(t\omega) + d \sin(t\omega)) \\ + \frac{1}{10}(-c \cos(t\omega)\omega^2 - d \sin(t\omega)\omega^2) = 0.$$

we can combine terms, simplify, and determine that the amplitude of the resulting oscillation of mass is

$$\text{amp}(\omega) = 10 \left| \frac{\omega^2 - 9}{100\omega^4 - 1809\omega^2 + 8100} \right|$$

A picture is worth How many words . . . or equations?

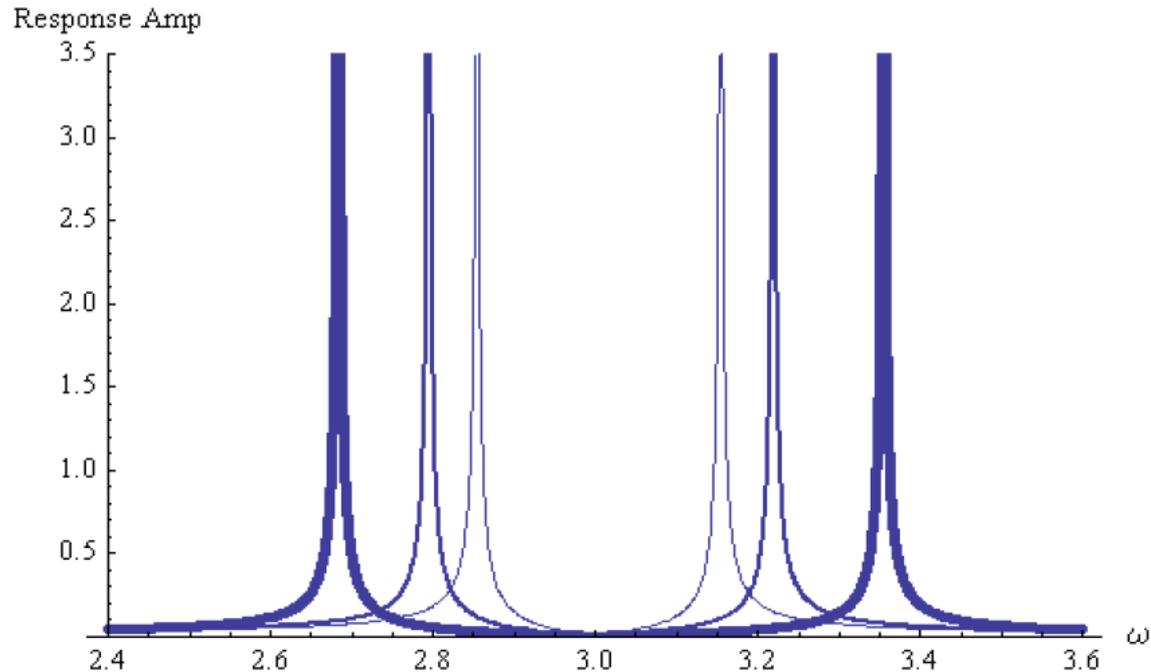


Figure 9: In case neither mass m_1 nor m_2 has damping, i.e. $c_1 = c_2 = 0$, here are plots of the response amplitudes of the primary system mass, m_1 , as a function of the driver frequency ω . We see in the three plots that as the ratio $\frac{m_2}{m_1}$ of added secondary mass m_2 as a percentage of the primary mass, m_1 , goes from 1%, to 2%, to 5%, (corresponding to the thin, thick, and thicker plots, respectively) the frequency region of low responses to the driver frequency surrounding the natural frequency of the structure expands.

So if we are willing to add more and more mass we get a wider “safe” region.

Incidentally, if we go back to just our one mass system

$$m \cdot y''(t) + c \cdot y'(t) + k \cdot y(t) = f(t), \quad y(0) = y_0, \quad y'(0) = v_0$$

with a driving force $f(t) = F_0 \cos(\omega t)$

Here is the steady state (non-homogeneous) solution for Equation (4) with $c \neq 0$:

$$\text{ss}(t) = -\frac{F_0 m \cos(\omega t) \omega^2 + F_0 k \cos(\omega t)}{m^2 \omega^4 + c^2 \omega^2 - 2km\omega^2 + k^2} + \frac{cF_0 \sin(\omega t) \omega}{m^2 \omega^4 + c^2 \omega^2 - 2km\omega^2 + k^2}.$$

The amplitude of the steady state solution as a function of input frequency, ω , is given in Equation (5).

$$\text{amp}(\omega) = F_0 \sqrt{\frac{1}{m^2 \omega^4 + c^2 \omega^2 - 2km\omega^2 + k^2}} = F_0 \sqrt{\frac{1}{c^2 \omega^2 + (m\omega^2 - k)^2}}. \quad (5)$$

How big can the response, $\text{amp}(\omega)$, really be?

We seek the maximum frequency response of $\text{amp}(\omega)$. Again, a picture

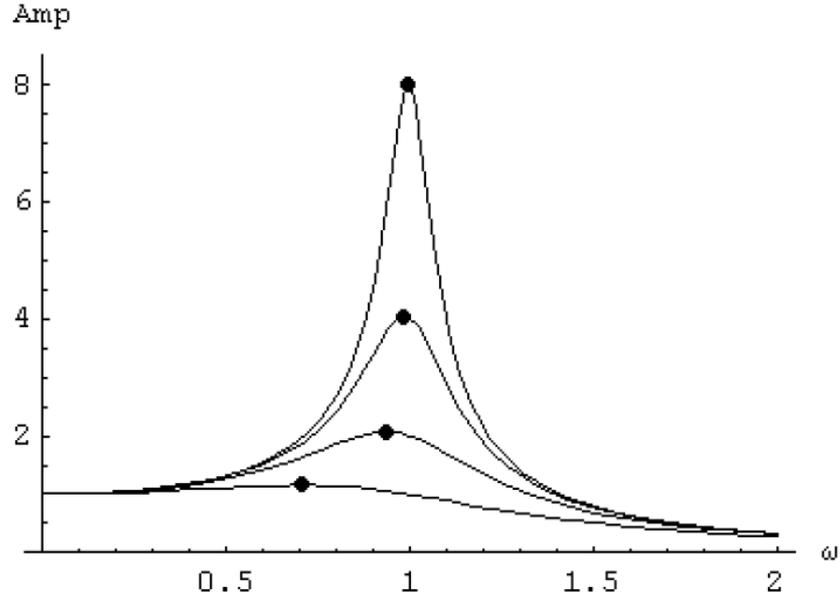
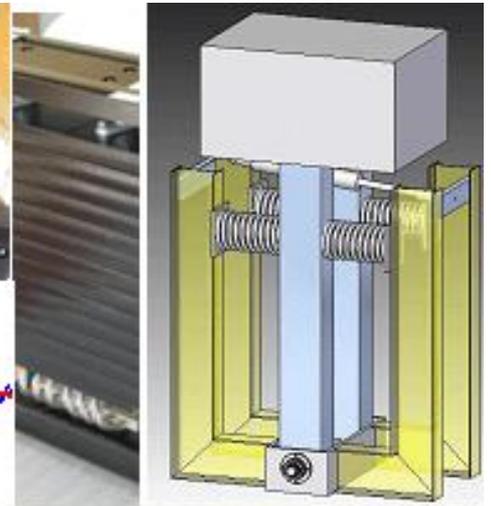
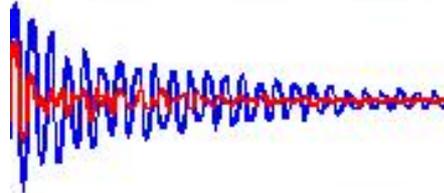
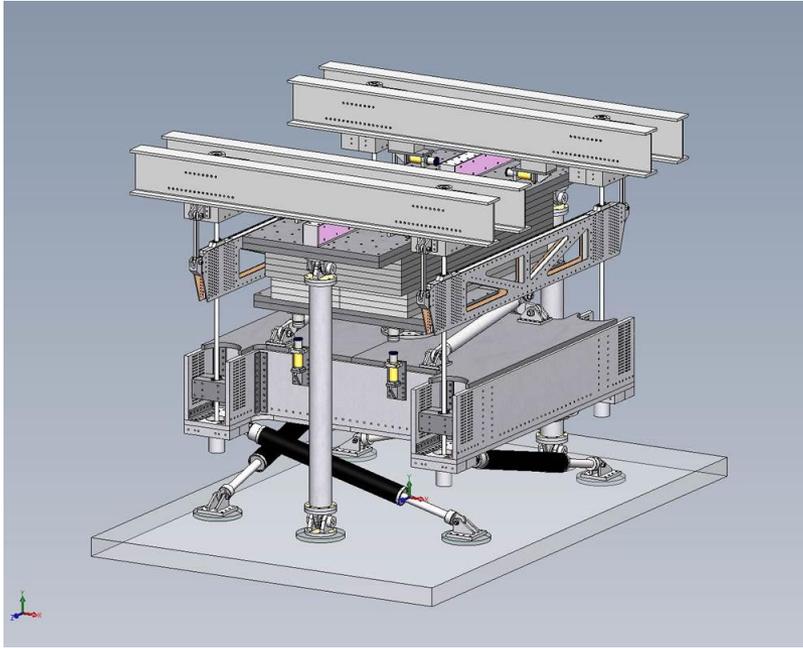


Figure 3: Plot of peak frequency response, both input frequency, ω , and amplitude, of steady state solution for $m = 1$, $k = 1$, $\omega_0 = 1$, $F_0 = 1$, for $c = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ where peak is highest for lowest value of c , i.e. $\omega_{\max} \rightarrow \omega_0$ as c decreases. These values appear in Table 1.

c value	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
Peak response frequency, ω_{\max}	0.707107	0.935414	0.984251	0.996086
Maximum response amplitude	1.1547	2.06559	4.03162	8.01567

Table 1. Peak frequency response for Equation (4) with $m = 1$, $k = 1$, $F_0 = 1$, for $c = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$.









THE HANCOCK AT 30 BEHIND THE LOOKING GLASS

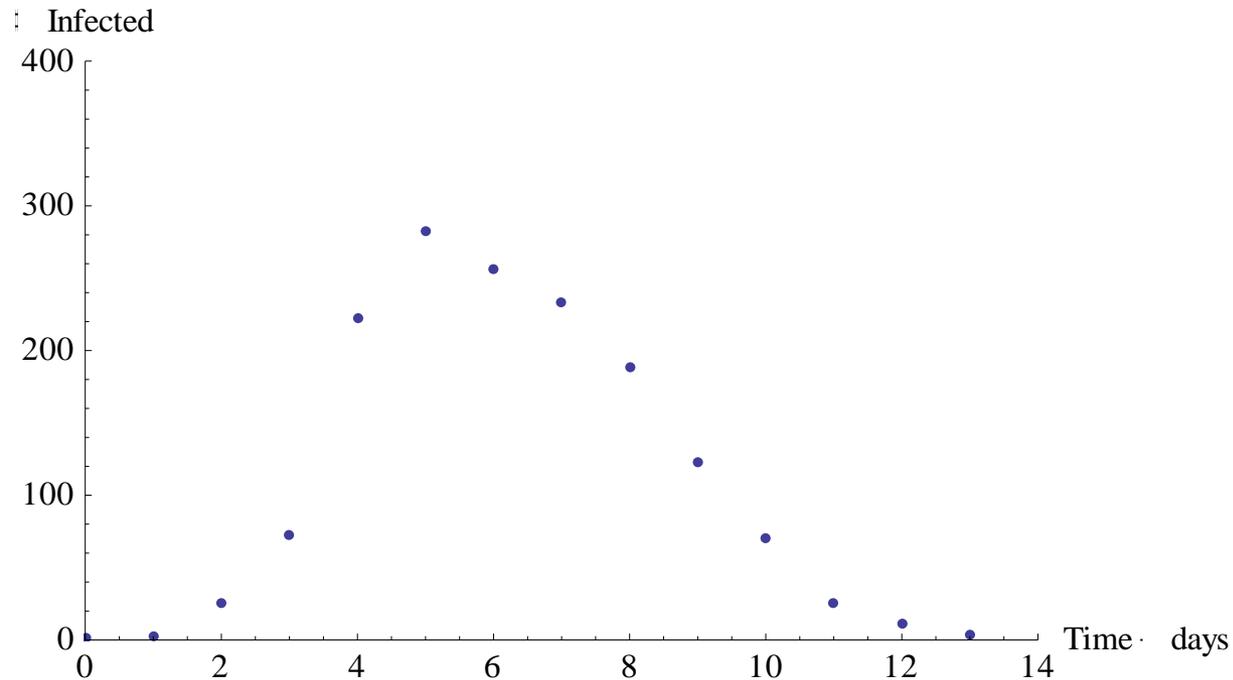
The Boston Globe boston.com



The tuned mass dampers

Time-days	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Infectives	1	3	25	72	222	282	256	233	189	123	70	25	11	4

Table 1. Total Number of Bedridden Boys on Day t .



Source: Communicable Disease Surveillance Center. 1978. News and Notes: Influenza in a Boarding School. *British Medical Journal*. 1(6112). <http://www.pubmedcentral.nih.gov/picrender.fcgi?artid=1603269&blobtype.pdf>. Accessed 5 September 2008.

Continuous Model

$$S'(t) = -aS(t)I(t)$$

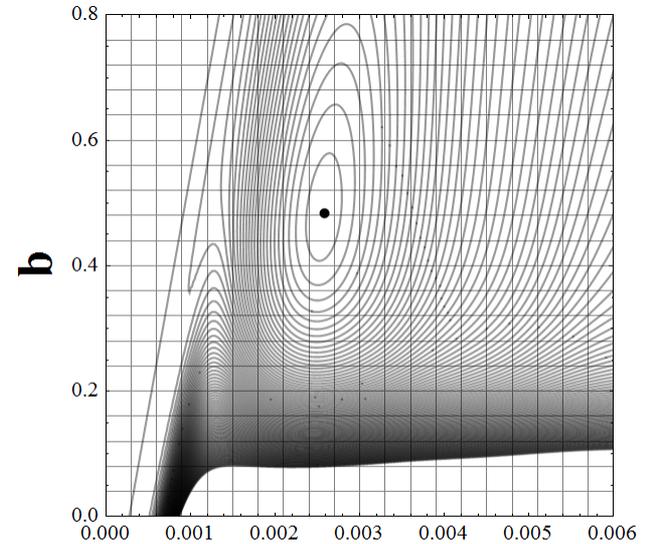
$$R'(t) = bI(t)$$

$$I'(t) = aS(t)I(t) - bI(t)$$

Using grid in parameter space to minimize

$$\text{SSE} = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

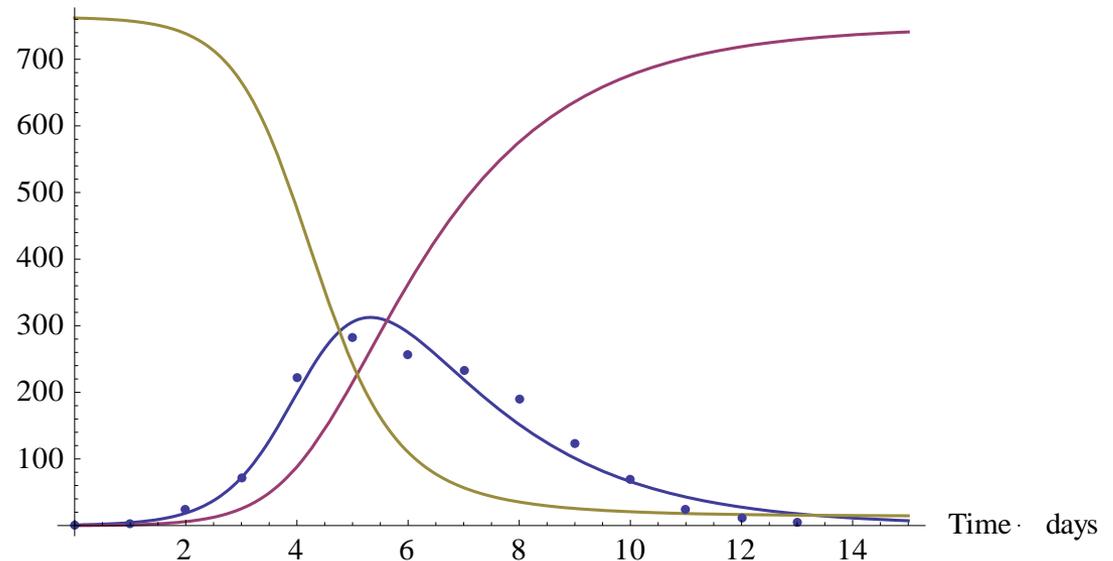
with $S(0) = 763-1$, $I(0) = 1$, and $R(0) = 0$ we obtain $a = 0.00218$ and $b = 0.441$.



a

F · Blue, S · Green, R · Purple

Populations



$$S(n + 1) = S(n) - aS(n)I(n)$$

$$R(n + 1) = I(n) + bI(n)$$

$$I(n + 1) = aI(n) + S(n)I(n) - bI(n)$$

Discrete Model

Using EXCEL's Solver to minimize

$$\text{SSE} = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

with $S(0) = 763-1$, $I(0) = 1$, and $R(0) = 0$ we obtain $a = 0.00291$ and $b = 0.72$.

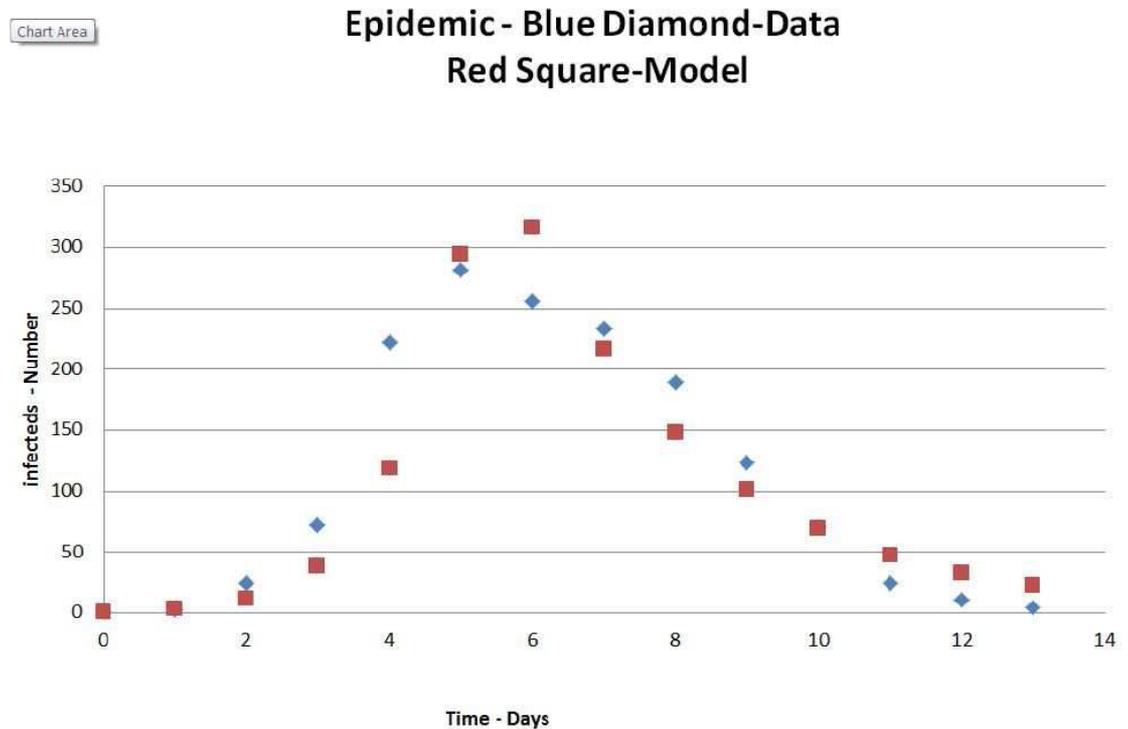
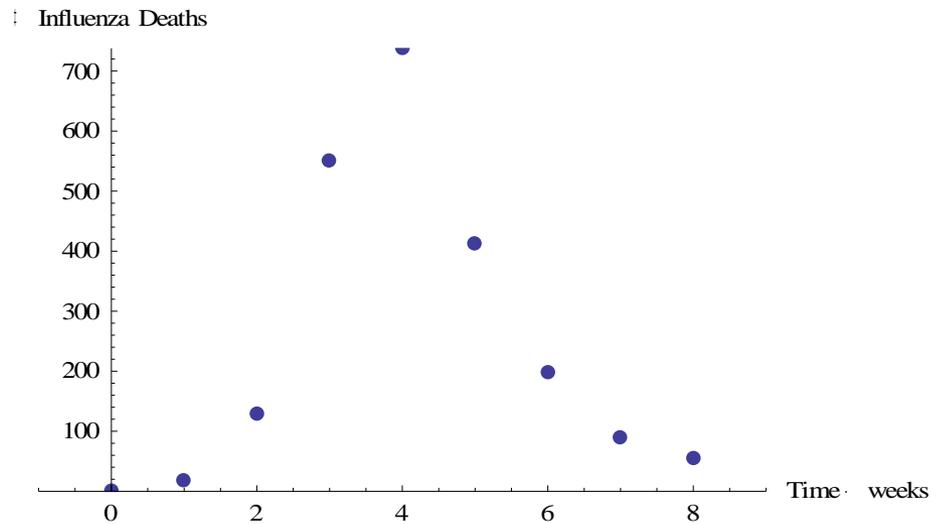


TABLE 1A
WEEKLY REPORTS FROM SAN FRANCISCO TO THE CALIFORNIA STATE
DEPARTMENT OF PUBLIC HEALTH

For the week ending	Cases	Influenza	Deaths Pneumonia	Total
Oct. 5	36			
12	531			
19	4,233	88	42	130
26	8,682	494	58	552
Nov. 2	7,164	679	59	738
9	2,229	376	38	414
16	600	174	24	198
23	164	72	18	90
30	57	42	14	56
Total		1,925	322	2,257
Dec. 7	722			50
14	1,517			71
21	1,828			137
28	1,539			178
Jan. 4	2,416			194
11	3,148			290
18	3,465			310
25	1,440			149

8-19



Source: Hrenoff, A. K. 1941. The Influenza Epidemic of 1918-1919 in San Francisco. *The Military Surgeon*. November: 805-811.

Source: Miller, S, and J. Helms. 2010. The Disease that Infected Half the World: Mathematics, Biology, and History. *PRIMUS*. 20(3): 245-260.

Collected data on falling coffee filters,
(time in s, position in m). Using Vernier
GoMotion detector.

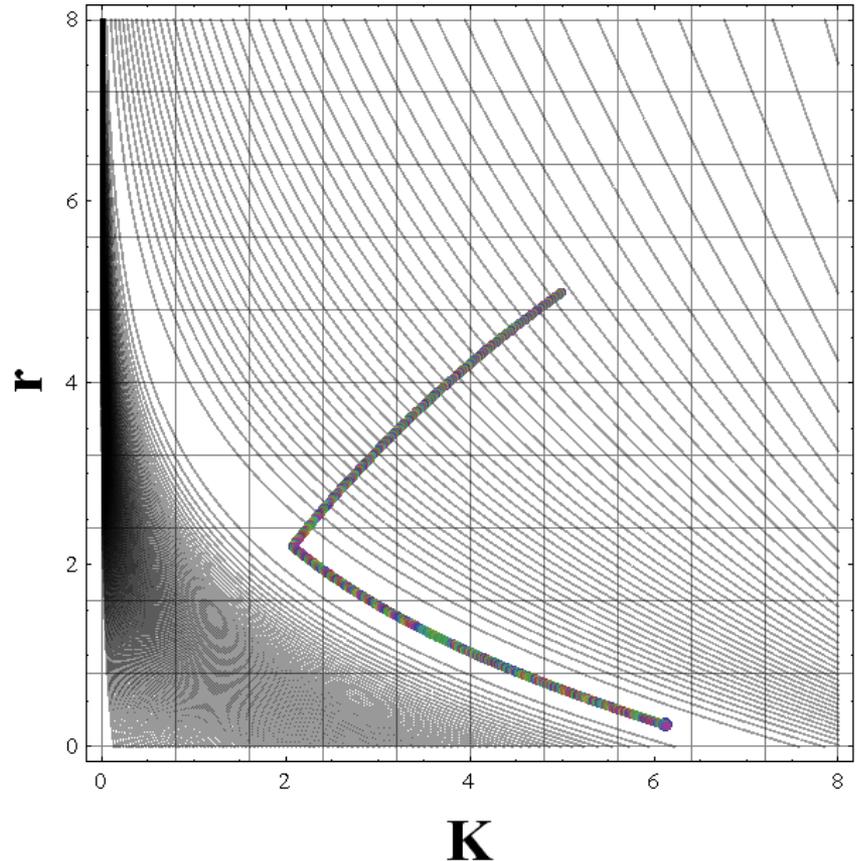
t	pos(t)
0.	0.
0.0313	0.042
0.0626	0.0873
0.0938	0.1353
0.1252	0.188
0.1565	0.2404
0.1878	0.2961
0.2191	0.3537
0.2504	0.4155
0.28177	0.4791
0.313	0.5445
0.3443	0.6095
0.3756	0.6844
0.4069	0.761
0.4382	0.8361

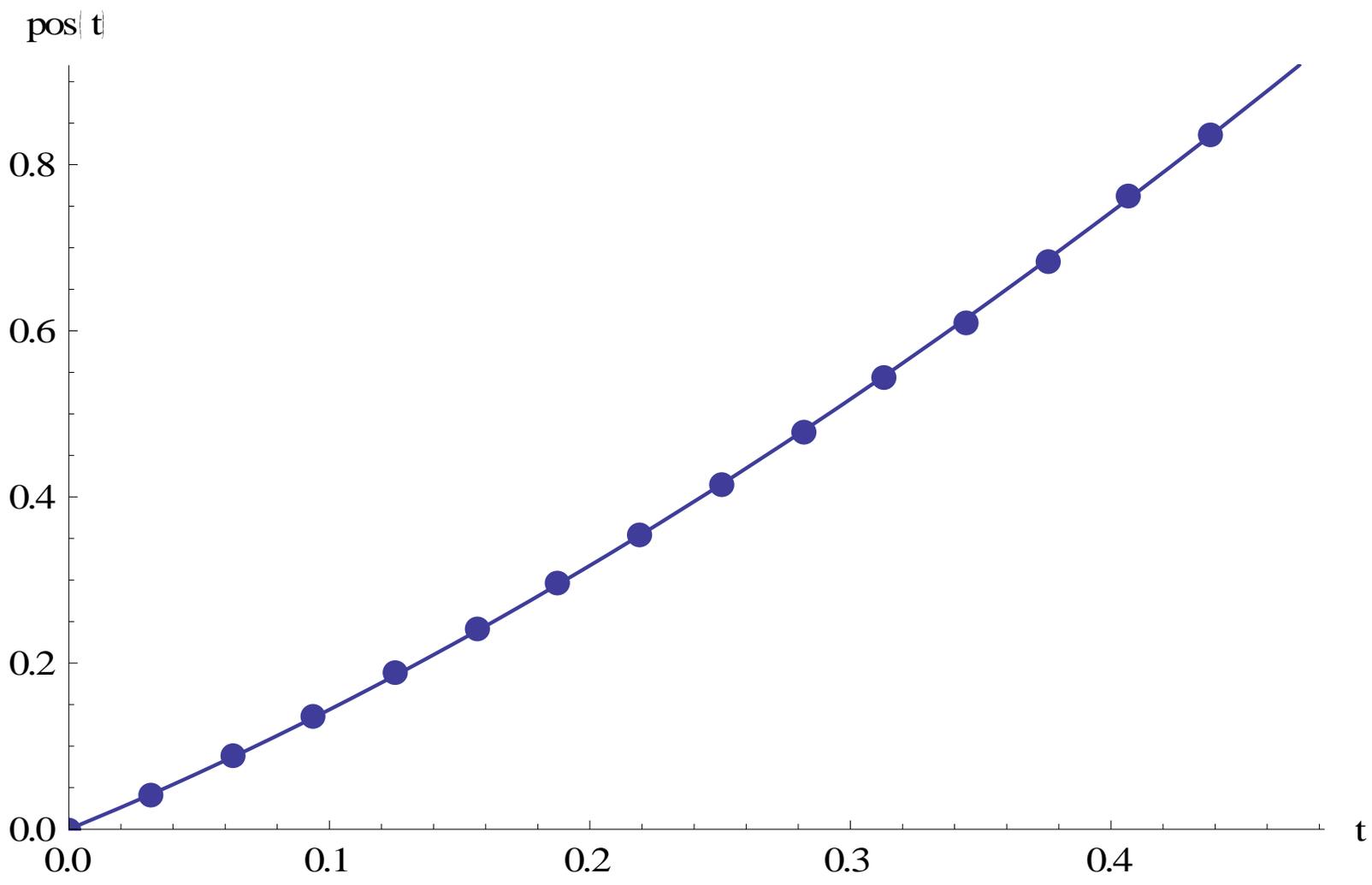
Grid in parameter space
(K, r, SSE(K,r)) and result of
gradient search.

Model built from Newton's Second Law

$$m y''(t) = m * g - k y'(t)^r, y(0) = 0, y'(0) = v_0$$

Usual cases $r = 1$ or $r = 2$. However, from data
 $k = 6.12$ and $r = 0.24$ with $SSE = 0.0000511$





The fit is

Spread of Disease Process

Experiment conducted at Cornwall NY USA by
DR Brian Winkel, PROF Emeritus US Military Academy,
West Point NY

Prepare a grid (print out M&MGridToUseForSimulations pdf file), mark 8 randomly selected cells as infected and write numbers 1 through 8 in each cell, respectively. Make a corral surrounding the grid so when we toss the M&M's they stay on the grid.

Source: Winkel, B. J. 2012. Sourcing for Parameter Estimation and Study of Logistic Differential Equation. *International Journal of Mathematical Education in Science and Technology*. 43(1): 67-83.

Place 54 M&M's in a cup and

- (a) gently toss them out onto the grid,
- (b) if an M&M contacts an infected cell/M&M mark the cell with next number of infected, first 9, then 10, then 11, etc. and remove infected M&M's from population,
- (c) when all M&M's are marked from this toss or generation collect M&M's and toss again until all M&M's are infected.

If you cannot perform this simulation then we have done this for you and enclose 12 generations of our simulation from which you are to gather data on the number of infected at each generation or time.

Equipment:

1 small bag regular M&M's

1 small cup

1 level surface

Early actions:

Prepare grid with 8 randomly selected infected cells. Number them as seen here.

Open bag and count out 54 M&M's.

Place M&M's in cup.

Gently toss M&M's onto surface.

If M&M contacts an infected cell/M&M mark the cell with next number of infected, first 9, then 10, then 11, etc.

Remove infected M&M's from population.

When all M&M's are marked from this toss

Or generation. Collect M&M's and toss again.



Later action:

Yummy!!!

Collect data for each generation on $y(t)$, the number of M&M cells which are infected.

Plot the number of infecteds as a function of the generation (time).

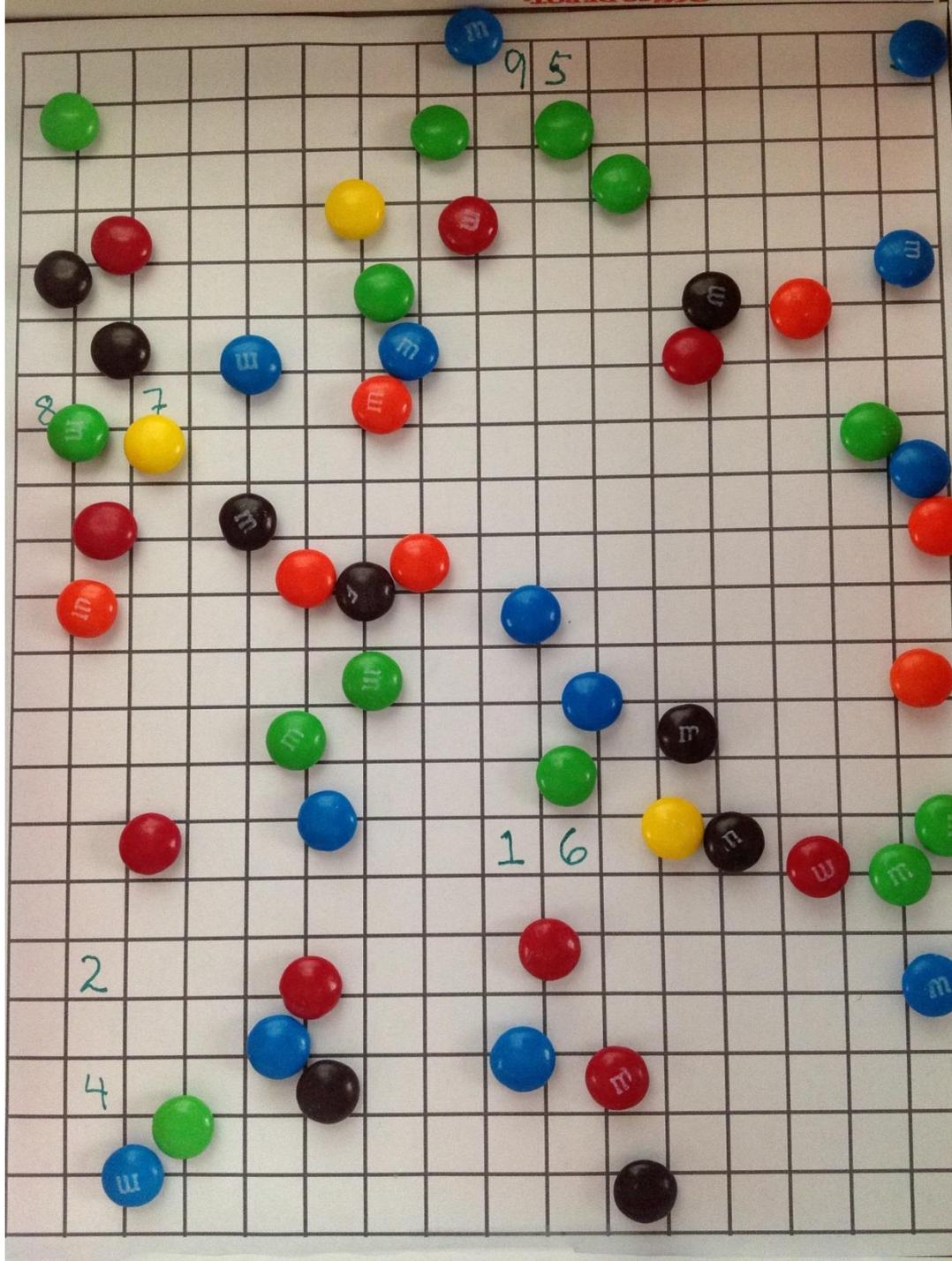
Propose a mathematical model, using a differential equation for the rate of spread of the disease.

Estimate the parameters using a sum of square errors method.

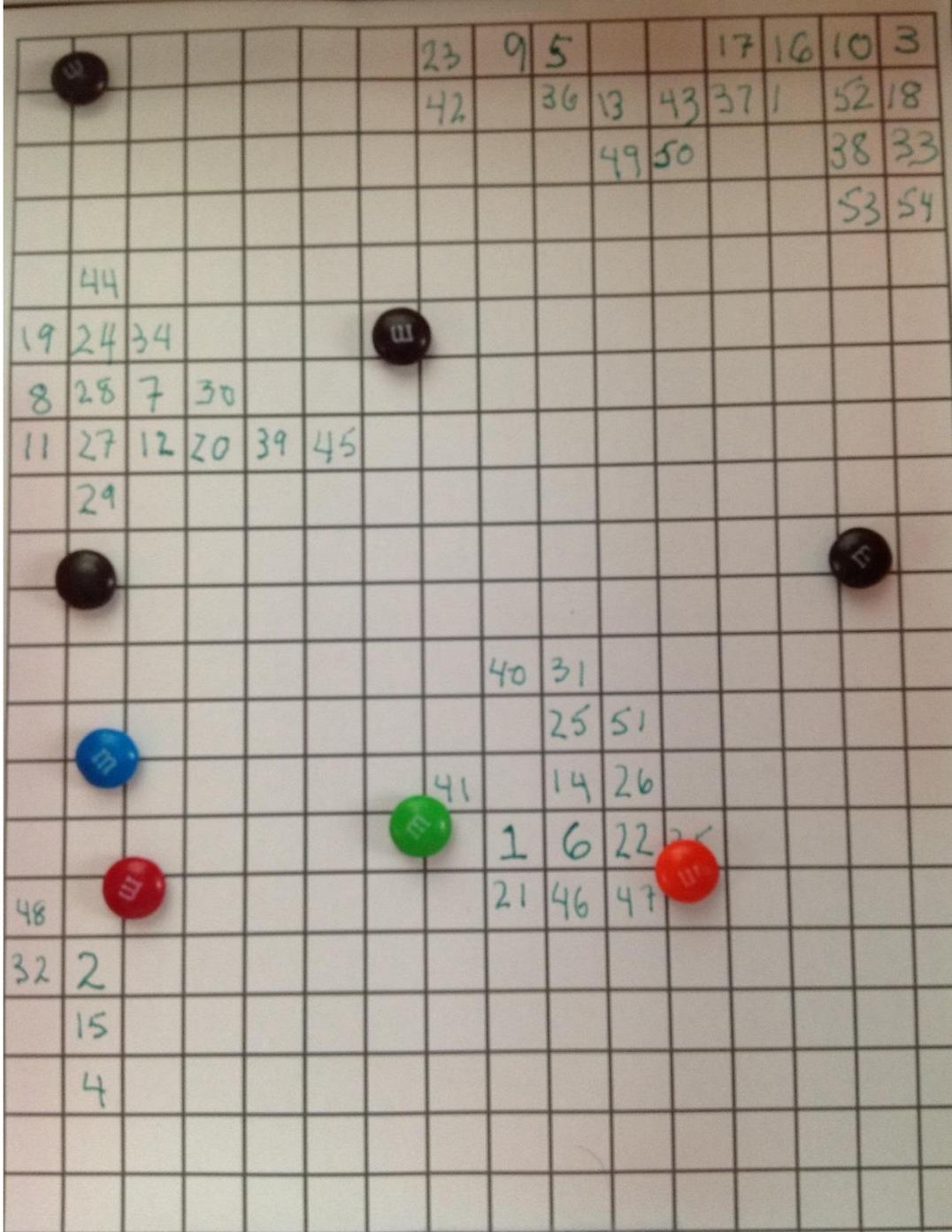
Compare your model to your data, numerically and graphically.

Does your model do a reasonable and reasoned job of modeling this data and hence this phenomenon?

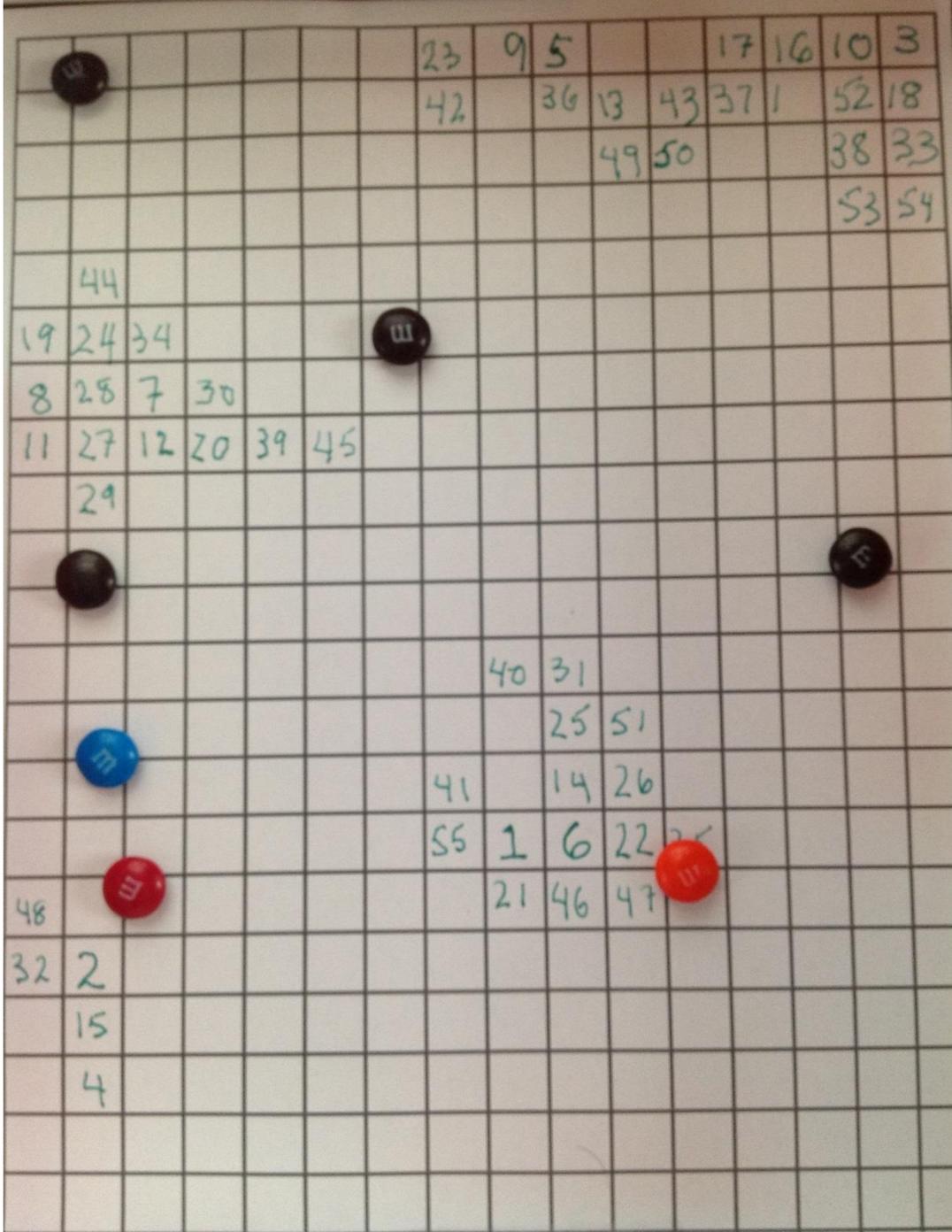
Generation 1



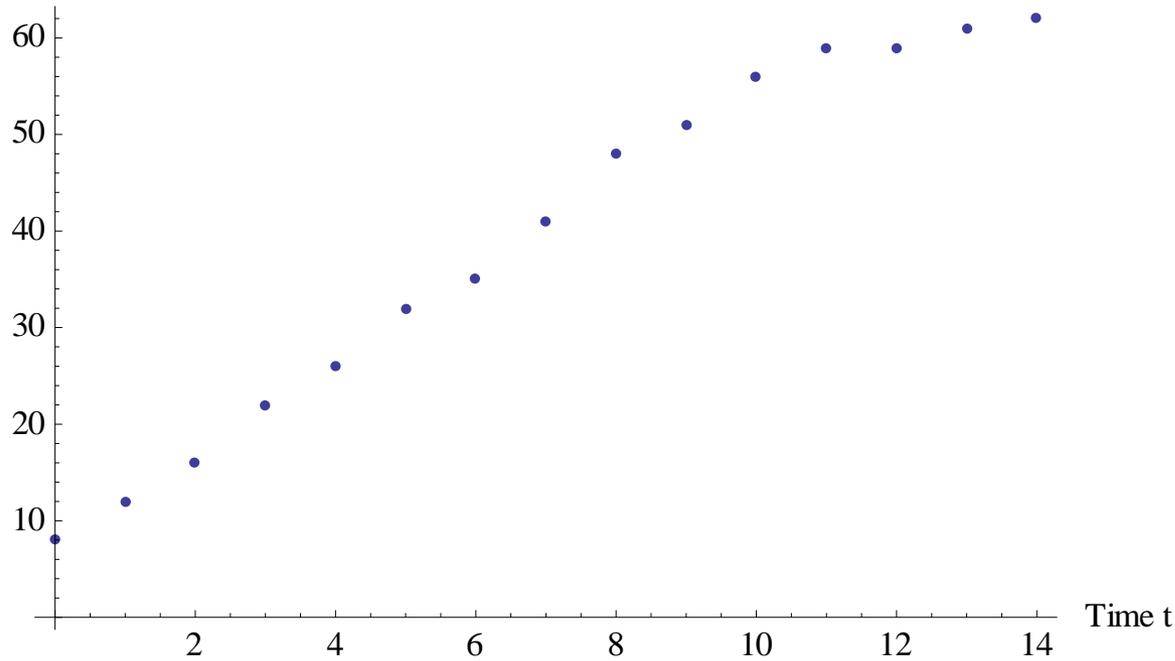
Generation 10



Generation 10



Population $y(t)$



Plot of data from M&M
Generation Efforts.

Logistic mathematical model of spread of disease.

$$y'(t) = ry(t) \frac{K - y(t)}{K}, \quad y(0) = y_0$$

```
ys[t_] = y[t] /. DSolve[{y'[t] = r y[t] (K - y[t]) / K, y[0] = 8}, y[t], t][[1]]
```

$$\frac{8 e^{r t} K}{-8 + 8 e^{r t} + K}$$

Solve DE and form SSE between Data and model.

```
SSE[r_, K_] = Sum[(data[[i, 2]] - ys[data[[i, 1]])]^2, {i, 1, Length[data]}]
```

$$\begin{aligned} & \left(12 - \frac{8 e^{r} K}{-8 + 8 e^{r} + K}\right)^2 + \left(16 - \frac{8 e^{2r} K}{-8 + 8 e^{2r} + K}\right)^2 + \left(22 - \frac{8 e^{3r} K}{-8 + 8 e^{3r} + K}\right)^2 + \left(26 - \frac{8 e^{4r} K}{-8 + 8 e^{4r} + K}\right)^2 + \\ & \left(32 - \frac{8 e^{5r} K}{-8 + 8 e^{5r} + K}\right)^2 + \left(35 - \frac{8 e^{6r} K}{-8 + 8 e^{6r} + K}\right)^2 + \left(41 - \frac{8 e^{7r} K}{-8 + 8 e^{7r} + K}\right)^2 + \left(48 - \frac{8 e^{8r} K}{-8 + 8 e^{8r} + K}\right)^2 + \left(51 - \frac{8 e^{9r} K}{-8 + 8 e^{9r} + K}\right)^2 + \\ & \left(56 - \frac{8 e^{10r} K}{-8 + 8 e^{10r} + K}\right)^2 + \left(59 - \frac{8 e^{11r} K}{-8 + 8 e^{11r} + K}\right)^2 + \left(59 - \frac{8 e^{12r} K}{-8 + 8 e^{12r} + K}\right)^2 + \left(61 - \frac{8 e^{13r} K}{-8 + 8 e^{13r} + K}\right)^2 + \left(62 - \frac{8 e^{14r} K}{-8 + 8 e^{14r} + K}\right)^2 \end{aligned}$$

Plot SSE = SSE(r, K) as contour plot.

We see roughly the values of r and K which could minimize our sum of square errors and hence give us our best fitting, in the sense of least square errors at around r = 0.35 and K = 65. This latter estimate is no surprise for our maximum number of M&M's in our population was 8 + 54 = 62, for the 8 originally infected and the 54 uninfected who eventually become infected in our simulation by touching squares/M&M's who are infected.

```
solrK = FindMinimum[SSE[r, K], {r, .35}, {K, 65}]
```

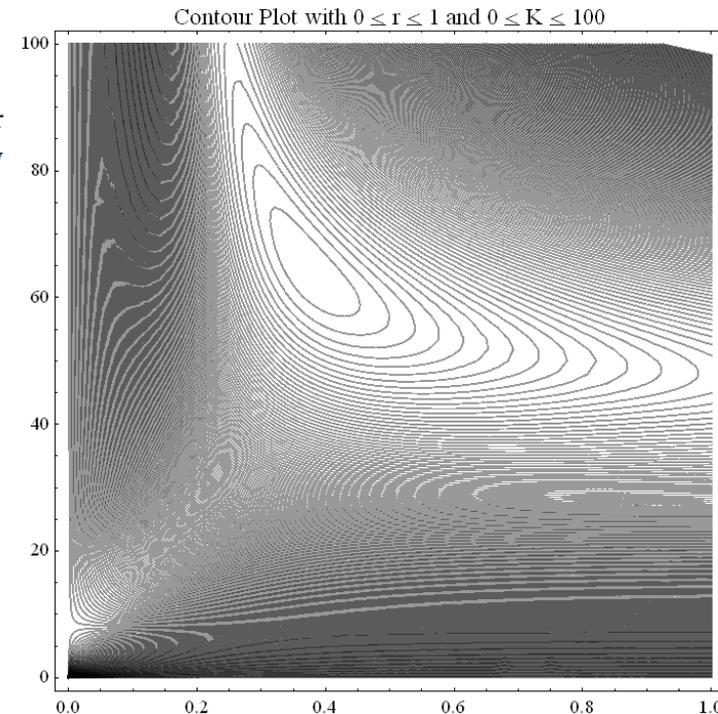
```
{19.9619, {r -> 0.37555, K -> 64.2538}}
```

We substitute our r and K values which make our sum of square errors between model and data into the model and plot this model on its own and then over the data to see how good our model is.

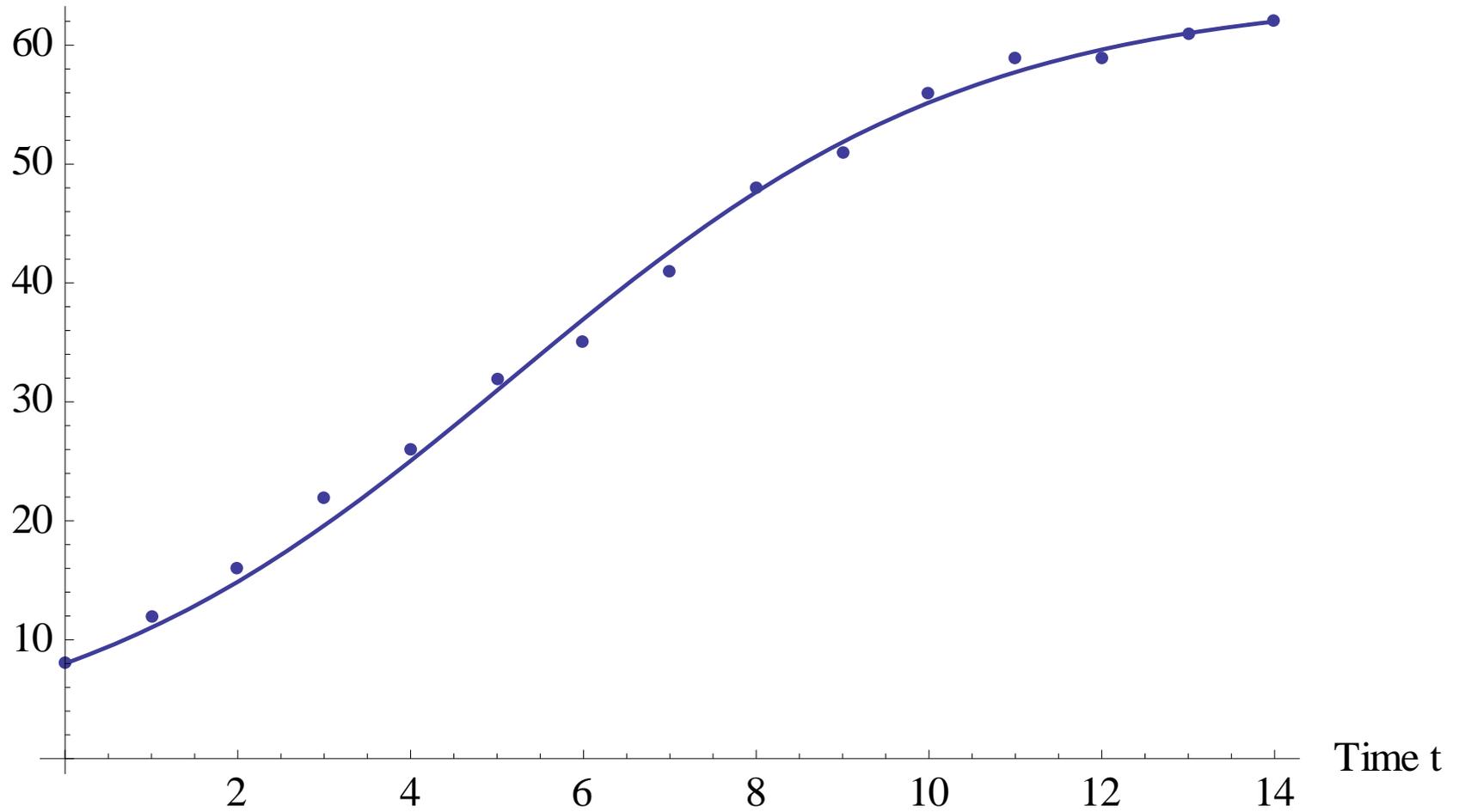
```
ysol[t_] = ys[t] /. solrK[[2]]
```

$$\frac{514.031 e^{0.37555 t}}{56.2538 + 8 e^{0.37555 t}}$$

```
ysolPlot = Plot[ysol[t], {t, 0, 14},
```



Population $y(t)$

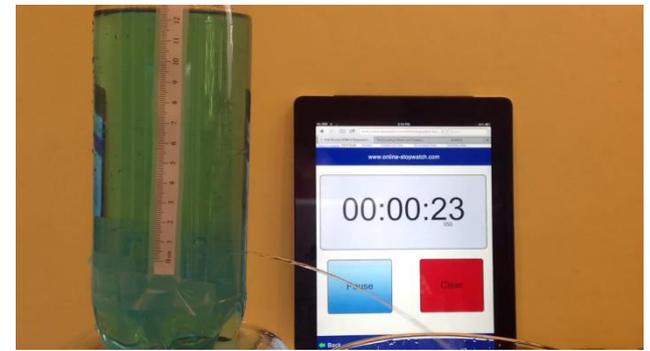


The fit is

[Torricelli's Law Video on SIMIODE YouTube Channel](#)

Build a mathematical model for height at time t ,
Say $h(t)$

$$h'(t) = \frac{a\alpha}{A} \sqrt{2gh(t)} \text{ which we got from } \dots$$

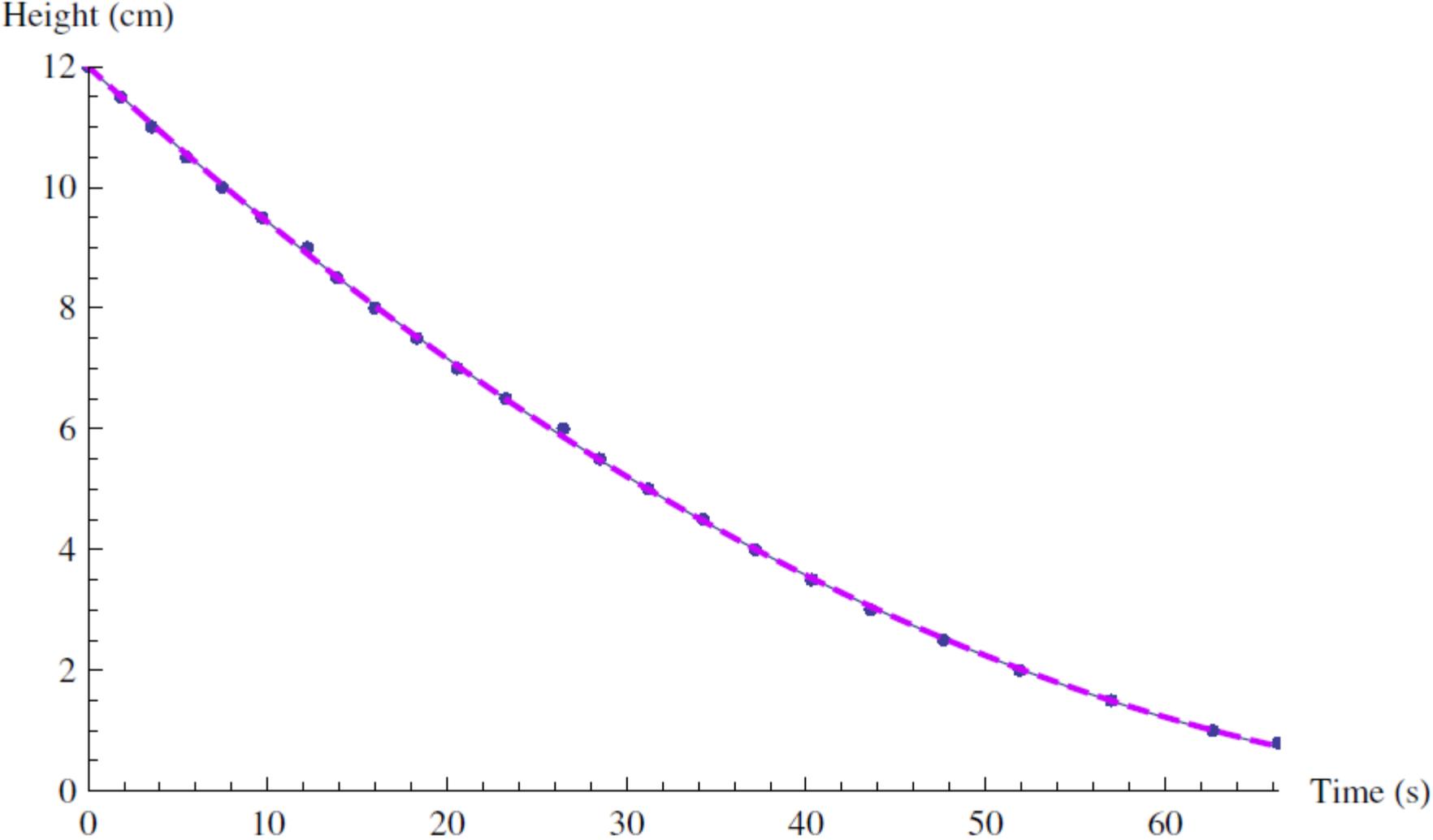


Torricelli's Law

$$A h'(t) = a \alpha \sqrt{2gh(t)}, \quad h(0) = y_0,$$

where A is the cross sectional area of the column of water - A was constant and the shape was circular in all Sources; g was the gravitation constant, presumed to be $9.80m/s^2$. In Sources 3-6 a is the cross sectional area of the small hole at the bottom of the column of water through which the water exits and $a = \pi r^2$ where r is the radius of the hole in bottom of the cylinder of water given in the title of the video, e.g., $13/16''$, $9/64''$, $11/64''$, and $7/64''$; α is the discharge or contraction coefficient, i.e., the effective percent of the tiny hole's area which actually permits water to flow out. In Sources 1 and 2 where the shape of the small hole at the bottom of the cylinder was now known we presumed that $a \alpha = k$, a constant.

Model compared to the data we collect...



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Each instructional unit has a Student Version with just STATEMENT while inside the Teachers Group there is a version with STATEMENT and extensive COMMENTS Section as well as other resources, e.g., Excel or Mathematica file with solution strategies.

We show a sample of the variety of applications or models to motivate the study of differential equations.

Modeling Death and Immigration with M&M 's and Simulation

Saving for Child's College Education

Modeling the Spread of Oil Slick with Incomplete Data

Analysis of incomplete reaction data

Modeling Immigration in a Petri Dish

How long does it take an ant to build a tunnel of length x in soil?

Hang Time Modeling

Modeling the spread of ICU's in US Hospitals from 1958-1974

Chemical Kinetics Models -Zeroth, First, and Second Order Reactions

Sublimation of Carbon Dioxide

Modeling the Emptying of a Column of Water

Modeling drugs for anesthesiology

Which melts first, a sphere or cube of ice of the same volume?

Modeling dissipation of intraocular gas bubbles used in eye surgery.

Changing temperature of container of water in a changing environment

Modeling sales of consumer products from a classic marketing study

Using data on whiffle ball fall model resistance and predict the fall position

Modeling a falling shuttlecock

Modeling Dialysis Machine

Modeling drug absorption of analgesic pain reliever under two different diets

Insect Colony Optimal Control

Modeling the amount of shampoo in a bottle during a shower

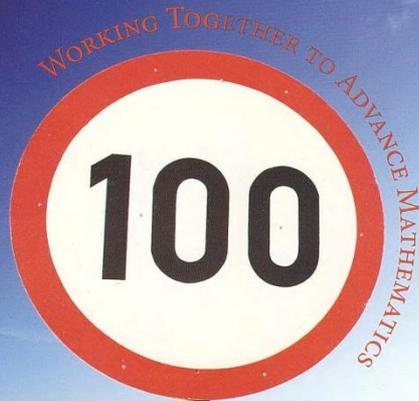
Building a model to help cross a river with current to land at specific spot

Convolution Applications – Machine Replacement Theory

PREP



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MATHEMATICAL ASSOCIATION OF AMERICA
CELEBRATING A CENTURY OF ADVANCING MATHEMATICS

Building Community in SIMIODE—Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

Brian Winkel, Jessica Libertini, Karen Bliss

July 19–25, 2015
Helena, Montana

REGISTRATION FEE: \$325 *per person*
(*\$400 for registrations received after*
June 7, 2015)

SIMIODE is a web-based community, combining the benefits of online-learning communities, teacher repositories, problem-based and inquiry-based learning, and technology into a web-based environment for the teaching and learning of differential equations through mathematical modeling. This workshop is for faculty who want to teach differential equations using the SIMIODE community. During the preparatory component, participants will join the SIMIODE community at www.simiode.org and experience Modeling Scenarios from a student viewpoint, Discussion Activities about important issues, and Teacher Groups for developing materials. During the onsite component, participants will have opportunities to engage as student groups, to realize the potential of SIMIODE, and to share reactions, experiences, and aspirations for engaging students. Colleagues will work in groups to begin the process toward publishing in SIMIODE. After the workshop activities, conversations will continue through SIMIODE's web features (e.g., Discussions, Projects, Newsletter) as PREP participants form their own special group inside the SIMIODE Teachers Group.

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SIMIODE

Systemic Initiative for **M**odeling Investigations
and **O**pportunities with **D**ifferential **E**quations

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Thank you.