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SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

STUDENT VERSION
PURSUIT MODELS

Brian Winkel
Director SIMIODE
Cornwall NY USA

STATEMENT

Consider these situations:

1. You are a driver for a *National Geographic* photographer on the Serengeti Plain. The photographer is gathering footage for a TV special on the cheetahs in the area. She wants to shoot footage of the cheetah chasing its prey. The real chase, as far as the driver is concerned, is getting close to the cheetah and keeping close. The prey of the cheetah is zigging and zagging and, as such, the cheetah is taking an unpredictable path so the driver needs a strategy to get close and stay close.
2. Johnny, your ex-best friend in third grade snatched your lunch box (containing the bag of cookies your Mom made!) and he is running about the school yard, humiliating you. You want to catch him in the worst way. You know he can run about the same speed as you. What strategy do you employ to chase him down? It is a huge school yard with flat open fields beyond.
3. Your soccer team is losing 2-1. In an end-of-game time kill situation, your opponents' best skilled player on the soccer team is on the field, handling the ball very nicely. You are determined to run him down and begin pursuing him. What is your strategy?
4. You are a wolf in open tundra and the pack has retired, but you are determined to bring home the kill. You see a small deer and it sees you. Like a flash the deer starts running, seemingly in a random path. What strategy do you use to track the deer down before you get exhausted from the chase?
5. You are driving a tank in dessert warfare and have isolated the enemy's command tank. You set out to chase down this special tank. What is your strategy?

Activities

1) Discuss what these situations have in common and produce several strategies the individuals involved might use. Be as precise as you can with each strategy and then compare the strategies as to merits and advantages. Also be sure to define the objective of the pursuer and how to know if and when the objective is achieved.
2) Suppose the object being chased is called the *evader* and the object doing the chasing is called the *pursuer*. Further, suppose the position of the evader is known by the evader only, in particular NOT by the purser, and is given by the mathematical expression $(x_e(t), y_e(t))$ for time t in seconds on the flat pursuit field. Distances can be in yards. Of course even the evader (as in the case of the cheetah), may not be determining a totally independent path of motion. However, to the pursuer it appears to be just the same as if the evader purposefully designed its path. Suppose now the pursuer's location is named (but not yet specified) as $(x_p(t), y_p(t))$ for time t in seconds on the flat pursuit field. What information would the pursuer have available at and leading up to time $t = T$ seconds into the chase? How could the pursuer use this information to mount a successful chase?
3) Use a sketch to describe your strategy as a pursuer to get close to the evader. Be sure to make your assumptions clear, e.g., what are your relative speeds? How long can you chase at this speed? One assumption we will offer is that the evader does not react to the pursuit. This might not be realistic. It presumes the evader is on a "fixed" if weavy course, but is not responding to the pursuer.

In Figure 1 we see the path or trajectory of an evader and in (1) we see the actual parametric equations which govern the evader's motion. Recall the pursuer does not know the evader's path; just where the evader is and where the evader has been!

$$\begin{aligned} xe(t) &= 5 + \cos(2\pi t) + 2\sin(4\pi t) \\ ye(t) &= 4 + 3\cos(3\pi t)\,. \end{aligned} \tag{1}$$

4) Consider the motion of the evader - the entity one is chasing, $(x_e(t), y_e(t))$ for time t in seconds. We could use the evader path offered in (1) and Figure 1 with units yards and seconds or one of your own choosing. Describe or model your pursuer's motion (based on some strategy you have designed), $(x_p(t), y_p(t))$, for time t in seconds so that the pursuer could have a chance of achieving the goals, e.g., getting close to the evader at some instance or staying close.

   Let us ask an easier question. Can you describe the rate of change of your motion, i.e. $(x_p'(t), y_p'(t))$ for time t? Is this the pursuer's speed AND direction? How does the evader's position play a role in your pursuit strategy? How do you implement that role in any equations you might construct? NB: Be sure you note the maximum speed speed$(t) = \sqrt{(x_e'(t))^2 + (y_e'(t))^2}$ of your evader and give your pursuer comparable speed so as to make this challenging, but not impossible. You could also investigate the situation with various pursuer speeds. For the evader

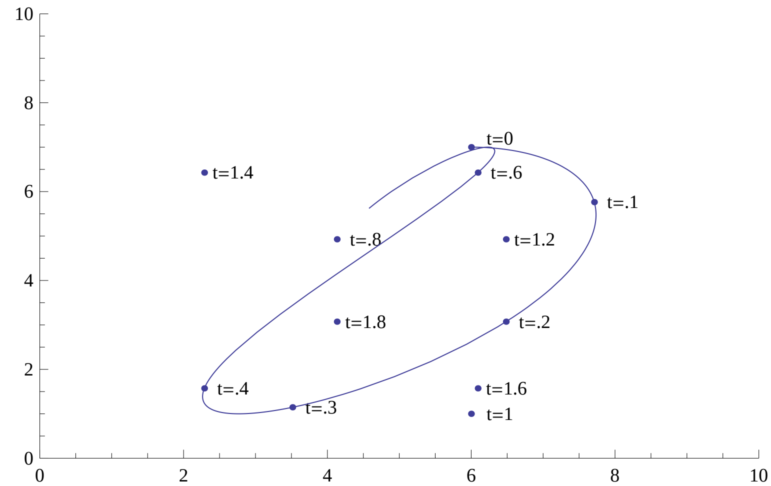

Figure 1. Plot of evader's path with times of positions in seconds and distances in yards displayed.

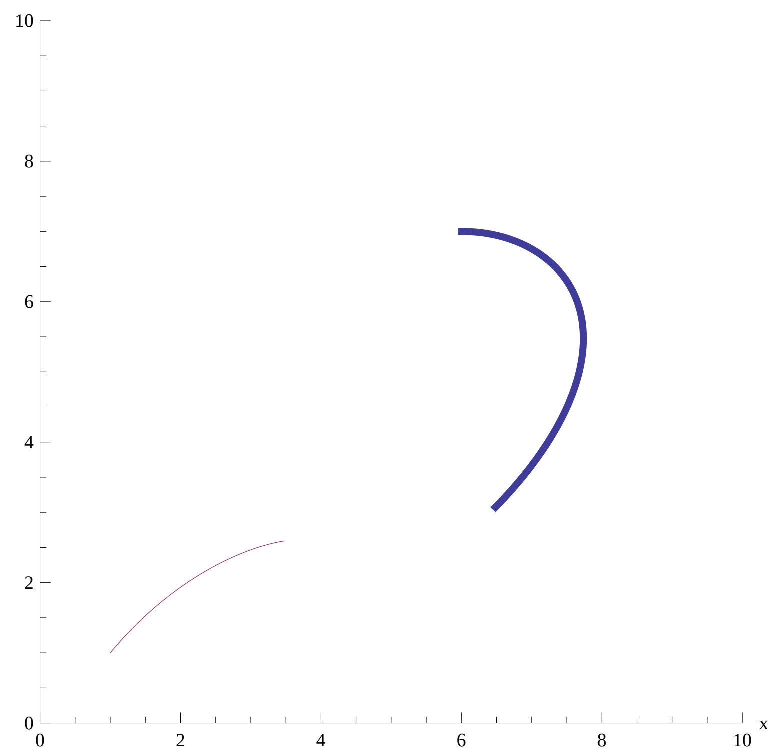

Figure 2. Plot of evader (thick, blue) and pursuer (thin, red) path with times of positions in seconds and distances in yards displayed. Here pursuer is always heading directly at where evader is at the moment.

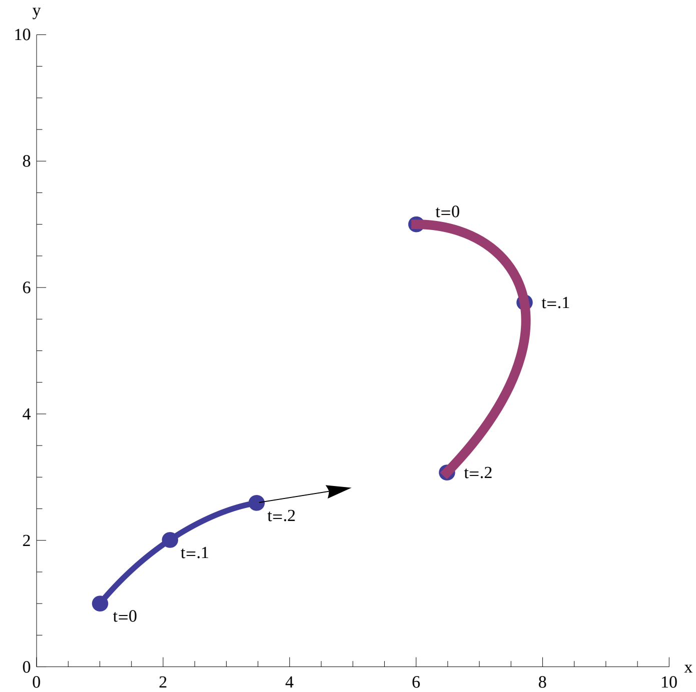

Figure 3. Plot of evader (lower left) and pursuer (upper right) paths with times of positions in seconds and distances in yards displayed. The direction in which the pursuer heads at time $t = 0.2$ is directly toward the location of the evader at time $t = 0.2$.

offered in (1) the maximum speed is 38.6 yards/second or 78.9 miles/hour. This is realistic for a cheetah over a very short course. You might also want to examine the maximum acceleration of your evader and of your pursuer, once you construct the latter's model? What are these in this case, i.e. what is maximum speed and maximum acceleration for evader and pursuer?

5) With one hand trace out an evader's path on a flat surface and with your other hand trace out a pursuer's path according to one of your strategies in "real" time, both at once. This is like patting your head and rubbing your tummy!

6) One strategy is for the pursuer to head directly at where the evader IS at time t. If you have not implemented this in response to (4) then do so now and compare your previous results with the ones from this activity. What criteria do you use to compare strategy results?

7) If the pursuer heads for where the evader IS (albeit there is a moving target at the evader's location) then perhaps by the time of arrival the evader will be gone, so might the pursuer do something else? Hint: In basketball and soccer one passes the ball NOT to where the intended receiver is, but to where one perceives the intended receiver will be. Just how long is this lead distance or time? How far should the lead be? Thus, construct a modification of activity (6) in which these considerations are used.

8) Pick an evader curve of your own. Keep it simple, like running round in a circle or running

along a straight line and see what your pursuer does using your strategies. Do starting positions matter? Are there any paths for the evader with starting positions for the pursuer which give obvious pursuer paths?