STATEMENT

Given two rectangular circular cylinders of water with the same volume, but different radii, with a small bore hole of same radius on the center of the bottom through which water exits the cylinder, which empties faster?

Recall, the differential equation, based on Torricelli’s Law[1, p. 3], for the height of a column of water in a constant cross sectional cylinder, $h(t)$, at time $t$:

$$A(h(t)) \cdot h'(t) = -a \alpha \sqrt{2gh(t)},$$  (1)

where $h(t)$ is the height of the column of water at time $t$; $A(h(t))$ is the cross-sectional area of the cylinder at height $h(t)$; $g$ is the acceleration due to gravity; $a$ is the cross sectional area of the bore hole at the base of the cylinder through which the water exits the cylinder; and $\alpha$ is an empirical number which indicates the percentage of the maximum flow rate which actually gets through the small bore hole. Usually $\alpha$ is on the order of 0.70. Note the negative sign on the right hand side of (1) as water is leaving the cylinder.

REFERENCES