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STUDENT VERSION
Design for First Passage Time

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STATEMENT

Consider the spring mass dashpot system described by the differential equation (1). Units are in MKS.

$$2 \cdot y''(t) + 4 \cdot y'(t) + k \cdot y(t) = 0\,, \quad y(0) = 2\,, \quad y'(0) = 0\,. \tag{1}$$

a) For what values of k will the system described in differential equation (1) be underdamped?

b) For each of the values of k in this range of values in (a) find the time the mass first has a displacement of 0. For each value of k call this time the *first passage time*, $FP(k)$.

c) Plot $FP(k)$ vs. k for $k \in [0, 20]$. What is happening for the value at $k = 0$ and $k < 2$ in general to this function?

d) For design purposes it may be required to offer up a spring with the lowest k value, but still have a first passage time of 0.5 s. Find the k for which $FP(k) = 0.5$.

Now consider the spring mass dashpot system described by the differential equation (2)

$$2 \cdot y''(t) + c \cdot y'(t) + 9.4 \cdot y(t) = 0\,, \quad y(0) = 2\,, \quad y'(0) = 0\,. \tag{2}$$

e) For what values of c will the system described in (2) be underdamped?

f) For each of the values of c in this range of values in (e) find the time the mass first has a displacement of 0. For each value of c call this *first passage time*, $FP(c)$. What is happening for the value of $c = 0$ to this function?

g) Plot $FP(c)$ vs. c for $c \in [0, 9]$.

h) For design purposes it may be required to offer up a spring with the lowest c value but still have a first passage time of 0.5 s. Find the c for which $FP(c) = 2$.

Here is a rather interesting result that seems to imply initial position has nothing to do with first passage time when all other values m, c, k, and v_0 are fixed!

i) For the motion described by (2) show that for a given value of c no matter what initial position y_0 given the first passage time $FP(c)$ is ALWAYS the same.

j) Also show the same is true in the case of the motion described by (2), i.e. for a given value of k no matter what initial position y_0 given the first passage time $FP(k)$ is ALWAYS the same.

Here are further considerations.

k) Incidentally, (i) does *not* apply to initial velocity, i.e. the first passage time does vary as we change $y'(0) = v_0$ if every other parameter stays fixed. Prove this result.

l) Neither does (j) hold true for mass m, i.e. the first passage time does vary as we change m if every other parameter stays fixed. Prove this result.