STATEMENT

Consider the spring mass dashpot system described by the differential equation (1). Units are in MKS.

\[ 2 \cdot y''(t) + 4 \cdot y'(t) + k \cdot y(t) = 0, \quad y(0) = 2, \quad y'(0) = 0. \quad (1) \]

a) For what values of \( k \) will the system described in differential equation (1) be underdamped?
b) For each of the values of \( k \) in this range of values in (a) find the time the mass first has a displacement of 0. For each value of \( k \) call this time the first passage time, \( FP(k) \).
c) Plot \( FP(k) \) vs. \( k \) for \( k \in [0, 20] \). What is happening for the value at \( k = 0 \) and \( k < 2 \) in general to this function?
d) For design purposes it may be required to offer up a spring with the lowest \( k \) value, but still have a first passage time of 0.5 s. Find the \( k \) for which \( FP(k) = 0.5 \).

Now consider the spring mass dashpot system described by the differential equation (2)

\[ 2 \cdot y''(t) + c \cdot y'(t) + 9.4 \cdot y(t) = 0, \quad y(0) = 2, \quad y'(0) = 0. \quad (2) \]

e) For what values of \( c \) will the system described in (2) be underdamped?
f) For each of the values of \( c \) in this range of values in (e) find the time the mass first has a displacement of 0. For each value of \( c \) call this first passage time, \( FP(c) \). What is happening for the value of \( c = 0 \) to this function?
g) Plot \( FP(c) \) vs. \( c \) for \( c \in [0, 9] \).
h) For design purposes it may be required to offer up a spring with the lowest \( c \) value but still have a first passage time of 0.5 s. Find the \( c \) for which \( FP(c) = 2 \).
Here is a rather interesting result that seems to imply initial position has nothing to do with first passage time when all other values $m$, $c$, $k$, and $v_0$ are fixed!

i) For the motion described by (2) show that for a given value of $c$ no matter what initial position $y_0$ given the first passage time $FP(c)$ is ALWAYS the same.

j) Also show the same is true in the case of the motion described by (2), i.e. for a given value of $k$ no matter what initial position $y_0$ given the first passage time $FP(k)$ is ALWAYS the same.

Here are further considerations.

k) Incidentally, (i) does not apply to initial velocity, i.e. the first passage time does vary as we change $y'(0) = v_0$ if every other parameter stays fixed. Prove this result.

l) Neither does (j) hold true for mass $m$, i.e. the first passage time does vary as we change $m$ if every other parameter stays fixed. Prove this result.