

SCUDEM

Student Competition Using Differential Equation Modeling

Sample Problem and Solution -- Comments on Solution

STATEMENT OF SAMPLE PROBLEM

Design an underwater table tennis game.



COMMENTS

In General

The competition requires a two page Executive Summary and a 10 minute Presentation.

An Executive Summary is typically a Summary of the results which is forwarded to an Executive for a decision. It might well be named Essential Summary as it should have the essentials of the activity described with attention to terms, definitions, assumptions, details, results or conclusions, and reflection, but NOT be laden with computations or reference material.

The 10 minute Presentation can be in any format, demonstration, PowerPoint slides, video, etc.

This is a competition about modeling in which differential equations are featured. This means modeling is at the heart of the activity and the tool for modeling is differential equations. It may be appropriate to present alternative modeling mathematics and compare several approaches.

Specific to this Sample Problem

We attach a one page Executive Summary and a 10 slide PowerPoint Presentation for the Sample Problem. Further we make comments on these submissions.

This Executive Summary describes the physical situation, sets out the rules of the table tennis game, and offers, using traditional physics knowledge, a defense of a second order differential equation for the velocity of the ball in the game. Terms and units are defined and made clear.

No solution techniques or even steps are offered, indeed, no solution is given either for velocity or position function of the ball. However, a description of what the solution implies is given with a depiction of the serve and first volley. Moreover, the comparison to traditional table tennis (out of the water) is the deepest reflection, as it indicates how the game has to be changed drastically in light of the resistance of the water to the motion of the ball. The mention of the “drop” of the steel ball from the loss of kinetic energy, due to the heavy resistance due to water is an observation, leading to some reflection.

The PowerPoint slides begin with an admission of the unexpected outcome due to the model, offering a reflective statement which compares the presumption about the motion of the ball in air to that in water and shapes the entire presentation. The notion of “Over and Out,” meaning just get it over the net and then bounce off the table, i.e. outside the table area, in order to facilitate a return the opposing player is the “Game changer.”

A Free Body Diagram is offered showing all the relevant force and Newton’s Second Law of Motion is cited for building the differential equation for velocity of the ball, $v(t)$, a vector quantity. The terms are explained completely and the units are given.

The use of an experiment to determine a parameter was an extra something offered in the process and then its use demonstrated the reasonableness in its value. The image of the serve and return volley demonstrates the reason for the rule that the ball must fall off the table in order to facilitate the return for there is little “bounce” of the ball once it reaches the opponent’s side.

The final fine rules with attention to equipment, a practical consideration, are good and the conclusions with improvements section suggest that there is room for improvements in the parameter estimation technique offered. The plot showing a model for the falling ball in the section which attempts to determine the parameter α could be faulted, for if one plotted the residuals they do not appear to be random about 0. There is an acknowledgement that this parameter is the only one which has to be estimated and the techniques used to estimate α is explained. Of course, there could also be another form for the resistance term, say proportional to velocity squared, but that is not addressed.

Finally, this solution does not offer other ideas, such as smaller table, a combination of in and out of the water, or higher propulsion from the racquet, for example. Although it does suggest that perhaps other materials might help both flight and bounce issues.

Because this submission does not consider or compare alternative physical configurations, transfers the resistance coefficient for rubber ball to steel ball without any consideration as to the correctness of doing so, and presents a rather difficult game we would suggest an Honorable Mention as a score. There is a complete, well presented model using differential equations and there is a presentation of the consequences of choices and physical laws.

We do not offer additional issues for Competition Saturday consideration but ask you, the reader, what generic additional issues you might raise which could be addressed, not necessary completed though, in a three hour morning session on Competition Saturday.