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STUDENT VERSION

Water Exit Bottle

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STATEMENT

We consider the rate at which a fluid drains out of a hole in a container. Our container is a cylinder made from a 2 liter plastic soda bottle.

Problem: A small hole is drilled in the side of a cylindrical container and the height of the water level (above the hole) goes from 10 cm down to 3 cm in 68 seconds. Estimate the height at intermediate times using the following two models.

1) The *linear model* where $h = h(t)$ is the height of the water in cm at time t in s:

$$\frac{dh}{dt} = k\,, \quad h(0) = 0\,, \quad h(68) = 3\,. \tag{1}$$

   Solve for $h(t)$.

2) *Torricelli's model* says that the time rate of change of volume V of water in the draining container is proportional to the square root of the water's depth h. This can be shown to yield

$$\frac{dh}{dt} = k\sqrt{h}\,, \quad h(0) = 10\,, \quad h(68) = 3\,. \tag{2}$$

   Solve for $h(t)$.

The table below gives the values of h predicted by each of these models for various times. You will fill in the column of observed values when we perform the experiment.

Time t	Linear h	Torricelli h	Observed h
0	10.0	10.0	
10	9.0	8.7	
20	7.9	7.5	
30	6.9	6.4	
40	5.9	5.4	
50	4.8	4.4	
60	3.8	3.6	
68	3.0	3.0	

3) Verify the values in the table from your solutions in (1) and (2), respectively.

4) Prepare a two-liter clear plastic soft drink bottle, whose midsection is essentially cylindrical, by drilling a clean 4 millimeter hole near the bottom of the cylindrical part. Attach to the bottle a strip of masking tape with centimeters marked on it and zero corresponding to the top of the hole. Perform the experiment, being sure to have a collection container for the water as it exits the cylinder. Fill in the last column in the table and compare with the two offered models, linear and Torricelli's.