

SCUDEM

Student Competition Using Differential Equation Modeling

Possible Problems

We pose several problem ideas to illustrate the nature of competition problems. We seek more and more ideas and problem candidates for promotion, for options in competitions. SCUDEM has a Problem Creation Team and welcomes ideas. Contact us at Director@SIMIODE.org.

1. Acorns, rodents, snakes

In northern forest there is a unique ecosystem consisting of northern timber rattlesnakes who eat forest rodents who in turn eat oak tree acorns. Be sure to include acorn masting. Model this ecosystem.

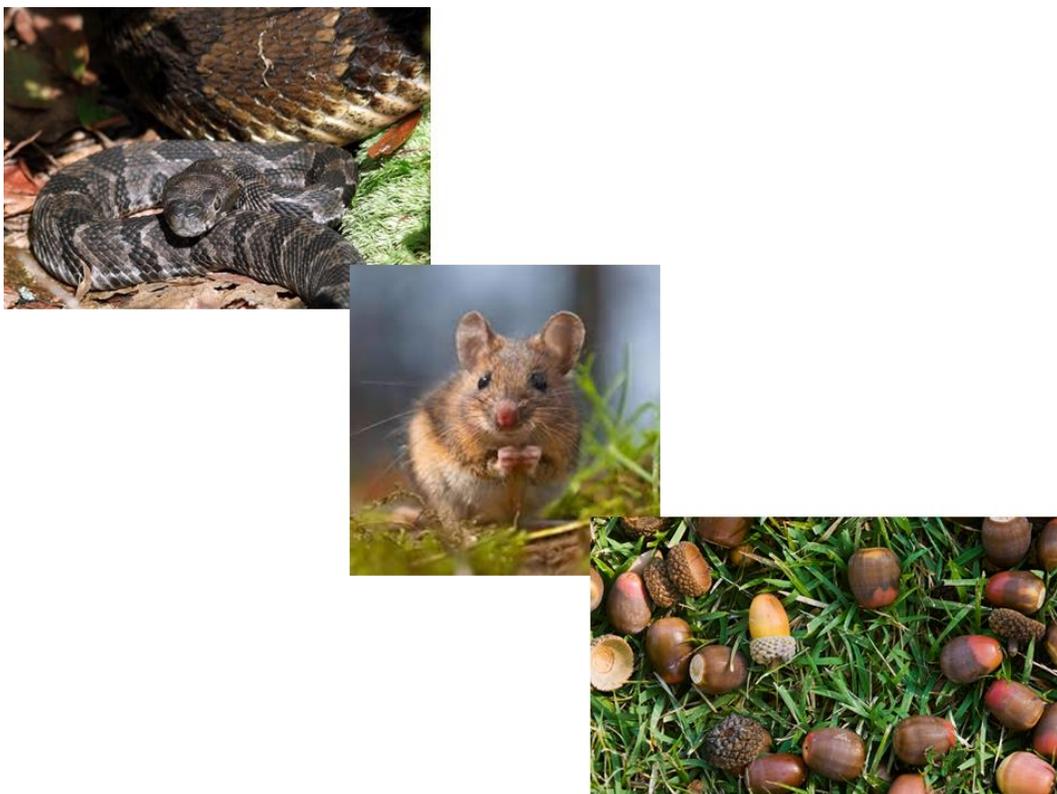


Figure 1. Northern timber rattle snake, forest rodent, acorns.

2. Silver Isotopes

If we knew the total mass of two silver isotopes (we are not sure these are ^{109}Ag and ^{110}Ag) (measured by the radioactive emissions - the more mass the more emissions) as a function of time could we determine what the initial amount of each was at the start, i.e. if we knew data on mass of silver present vs. time could we determine the initial amounts of each isotope AND could we confirm the decay constants or half-life of each isotope values, essentially validating a belief that these are indeed the two isotopes.



Figure 2. Silver Ingot.

3. Pendulum under water

Model the motion of a pendulum which sits entirely under water.

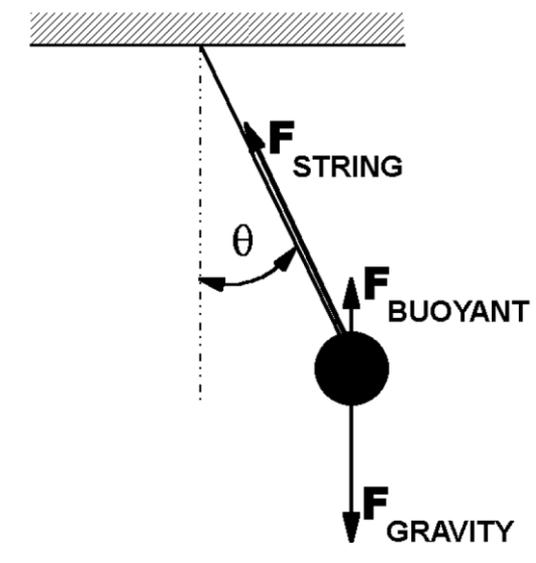


Figure 3. Free Body Diagram for UnderWater Pendulum.

4. Plant pollen in sediments

A. J. Craig (Absolute pollen analysis of laminated sediments: a pollen diagram from north-eastern Minnesota, M.S. thesis, University of Minnesota, 1970) counted pollen in a phenomenal 9400 sedimentary laminae from a core at Lake of the Clouds. According to his data, as white pine invaded the region surrounding the lake, it competed with entrenched populations of jack pine (*Pinus banksiana*) and red pine (*Pinus resinosa*), which occupy essentially the same coarse, sandy soil as *P. strobus*. The combined pine tree pollen accounted for about 60-70% of the pollen during the period of interest; other plant species remained essentially constant during the time period (with the exception of spruce (*Picea*) which decreased).

Craig's data is condensed and analyzed by Watts, R. A. 1973. Rates of change and stability in vegetation in the perspective of long periods of time. In *Quaternary Plant Ecology*. Birks, H. J. B. and R. G. West, Eds. Hoboken NJ: Blackwell Scientific, in Figure 1. In the first column, time is measured in units of thousands of years from 9131 years ago.



Figure 4. *Pinus resinosa* (L) and *Pinus banksiana* (R).

Years since 9131 years ago	% <i>P. Bank</i> / <i>P. Resin</i>	% <i>P. strobus</i>
0.000	53.4	3.2
0.259	65.5	0.0
0.640	61.8	3.7
1.010	55.2	3.4
1.410	60.4	1.7
1.769	59.4	1.8
2.126	50.6	10.6
2.432	51.6	7.0
2.687	40.0	21.2
3.148	29.7	34.2
3.618	25.0	40.4
4.109	32.5	29.8
4.613	22.7	46.2
5.029	31.6	33.0
5.507	32.5	37.6
5.963	27.1	39.5

Table 1. Data on *Pinus banksiana* and *Pinus strobus*.

Build a mathematical model of these competing species, estimate the parameters involved, and compare your model to the data.

5. Boeing Chinook Fuselage Study



Figure 5. Boeing CH-47-Chinook Helicopter over water.

In the manufacturing process of the Boeing CH-47-Chinook Helicopter there are five 3" diameter holes punched in the bottom of the fuselage for mounting processes along the main axis of the fuselage. These are replugged further along the manufacturing process. Boeing wants to know in the event of a water landing if the plugs falter and pop out will the helicopter sink? Build a mathematical model to address this issue.

6. Kidney Dialysis Machine

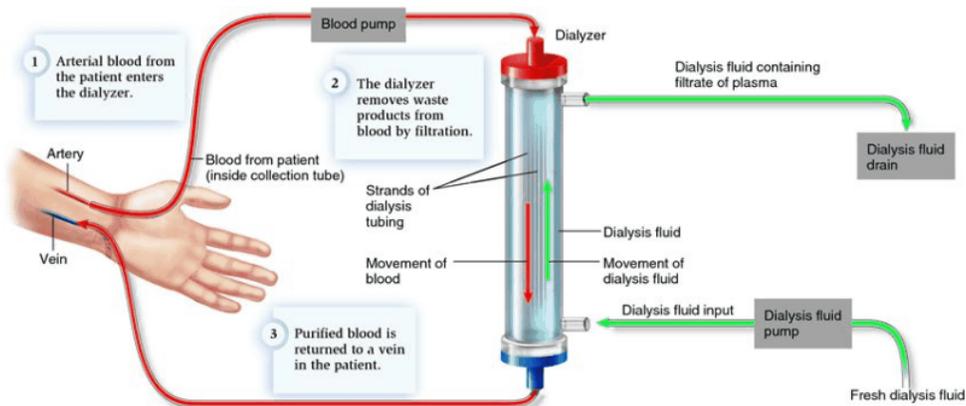


Figure 6. Schematic of Kidney Dialysis apparatus.

Design a simple (but realistic) kidney dialysis machine to permit medical students to see the comparison between counter and concurrent dialysis, the speed of the dialytic fluid compared to blood flow rates, the permeability of the membranes used, and the diffusion rate of impurities through the membranes separating the blood and the dialytic fluid.

7. Submersible Data Collector

Design an autonomous submersible device with no strings attached for collecting limnologic data which will sink to a reasonable depth in a fresh water lake and then return to the surface without any need to control or exert energy for the process.



Figure 7. SciFi depiction of Submersible Device.

8. Movie Revenues

Model revenue-cost stream for a major motion picture release from time of inception to 10 years after completion of distribution.



Figure 8. Movie marquee, representing just one way to generate revenue.