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SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

STUDENT VERSION

Toricelli Box

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STATEMENT

Torricelli's Law

We study the emptying a column of water through a small bore hole at the bottom of the column of water. We use Torricelli's Law [1] in which we have a differential equation (1) we can use to solve for the height, $h(t)$ m, of the water in the column at time t as the water is emptying the column through a small bore hole at the bottom of the column of water:

$$A(h(t))h'(t) = -\alpha a\sqrt{2gh(t)}, \tag{1}$$

where a is the area of the small bore hole at the bottom of the cylinder through which the water exits the cylinder, α is the restriction coefficient or percent of the water which could flow out which actually does flow out, and $A(h(t))$ is the cross sectional area of the water in the cylinder at height h.

Emptying Column of Water with Boxes of Varying Size at Base

We consider the problem of emptying a column of water through a small bore hole at the bottom of the column of water with a box on the bottom of the column, displacing water of volume equal to the volume of the box. Thus the water in the column will rise accordingly. We keep the Length (L) and Width (W) of the box the same in all cases, but vary the height, s, of the box.

We ask the question, "As the height of the box, s, increases how does the time to empty the column of water change?" Does the time to empty the column of water get more or less than when there is no box in the column of water?

Consider a right circular cylinder partially filled with a column of water to a height of h_i m. Let us suppose the cross-sectional shape of the column of water is a circle of radius r m. Place a rectangular solid (solid box) of dimensions L m long, W m wide, and of s m high, into the column of water so it settles on the bottom. It therefore displaces $L \cdot W \cdot s$ m^3 of water.

This box causes the water level in the cylinder to rise, but by how much? Call this height change h_c m. The volume of water displaced by the box in the water is $L \cdot W \cdot s$ m^3. Thus we have the original volume of water, $V_i = \pi r^2 h_i$, and we add the volume of displacement $L \cdot W \cdot s$ to get a total volume of material in the cylinder of radius r. This means we can determine the change in the height of the water, h_c by solving for h_c in $\pi r^2 h_c = LWs$, where $\pi r^2 h_c$ is the cylindrical volume of water displaced by the box.

Use Torricelli's Law appropriately and address these questions. As the height of the box, s, increases how does the time to empty the column of water change? Does the time to empty the column of water become more or less than when there is no box in the column of water?

Use the following variables: $\alpha = 0.70$, $a = \pi(.02)^2$ m^2, $r = 0.1$ m, $L = 0.06$ m, $W = 0.06$ m, $g = 9.8$ m/s^2, and $h_i = 0.6$ m.

Examine the cases where the height of the submerged box, s, is $s = 0.1, 0.2, 0.3, 0.4, 0.5,$ and 0.6 as well as the maximum height of the box where the top of the box is level with the surface of the water, for beyond that height a portion of the box is out of the water and that portion of the box which is out of the water does not alter our model. In each case, determine the time it takes to empty the column of water and compare that time to the time it takes to empty the column of water with no box in it.

Modeling the Emptying Column of Water with Box at Base Using Video Data

We provide two videos 1-064-Torricelli'sLawWithNoBox.mp4 and 1-064-Torricelli'sLawWithBox.mp4 in which we show the fall of the height of a column of water in a graduates cylinder, the first with no box at the base of the column of water and the second with a box at the base of the column of water.

We apply Torricelli' s Law for a Falling Column of Water to model the height of the column of water for a fixed graduated cylinder in two cases:

1. the graduated cylinder is filled to 500 ml (marked as a height of 23.6 cm) and water exits a small bore hole at the bottom which has a diameter of 5/32" or radius of 0.198438 cm
2. the graduated cylinder is initially filled to 500 ml and a configuration of four small blocks is placed at the bottom of the cylinder, thus raising the height of the column of water, now marked as a height of 26.25 cm. The water exits a small bore hole at the bottom which has a diameter of 5/32" or radius of 0.198438 cm.

The inner radius of the graduated cylinder in the videos is 2.6 cm.

Model both situations for the height of the column of water as a function of time using the differential equation (1) for Torricelli's Law and ascertain which column of water empties first, the column with the blocks or without the blocks.

REFERENCES

[1] Winkel, B. 2015. 1-015-S-Torricelli. https://www.simiode.org/resources/488.