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STUDENT VERSION
Cooling Up and Down

Brian Winkel, Director
SIMIODE, Cornwall NY USA

STATEMENT

In a very interesting (one of many great entries involving differential equations) blog [1] John D. Cook revisits one of his childhood questions, this one about heat.

> When I was a little kid, I asked some adults the following question.
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> If hot things cool, and cool things warm up, could something hot cool down and warm back up?
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> The people I asked didn't understand my question and just laughed. I have no idea how old I was, but I wasn't old enough to articulate what I was thinking.
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> Here's what I had in mind. I knew that hot things like a cup of coffee grew cold. And I knew that cold things, say a glass of milk, get warm. Well, could the coffee get so cold that it becomes a cold thing and start to warm back up?
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> Could the coffee become as cold as the glass of milk? Common sense suggests that can't happen. When we say coffee grows cold, we mean that it becomes relatively colder, closer to room temperature. And when we say the milk is getting warm, we also mean it is getting closer to room temperature. We've never left a hot cup of coffee on a table and come back later to find that it has cooled off so much that it is colder than room temperature. But could there be small fluctuations?
>
> As the coffee and milk head toward room temperature, could they overshoot the target, just by a little bit? Say room temperature is 70°F, the coffee starts out at 150°F, and the milk starts out at 40°F. We don't expect the coffee to cool down to 40°F or the milk to warm up to 150°F. But could the coffee cool down to 69.5°F and then go back up to 70°F? Could the milk warm up to 70.5°F and then cool back down to 70°F?

I didn't get a satisfactory answer to my childhood question until I was in college. Then I found out about Newton's law of cooling. It says that the rate at which a warm body cools is proportional to the difference between its current temperature and the ambient temperature. This law can be written as a differential equation whose solution shows that the temperature of a warm body decreases exponentially to the ambient temperature. The temperature curve always slopes downward. It doesn't wiggle even a little on its journey to room temperature. Cold bodies warm up the opposite way, exponentially approaching room temperature but never exceeding it.

In case this seems obvious, think about thermostats. They don't work this way. Say the temperature in a room is 85°F and you'd like it to be 72°F, so you turn on the air conditioning. Will the temperature steadily lower to 72°F? Not exactly. If you were to plot the temperature in the room over time and look at the graph from far enough away, it would look like it is steadily going down to the desired temperature. But if you look at the graph more closely, you'l see wiggles. The AC may cool the room to a little below 72°F, maybe to 70°F. The AC would cut off and the temperature would rise to 72°F. Unlike the cup of hot coffee, the AC will often overshoot its target, though not by too much. The temperature may feel constant, but it is not. It oscillates around the desired temperature.

Activities

1. Build a simple differential equation model of an air conditioner cooling down a small room, originally at 90°F, to a comfortable 65°F.
2. Make the model more realistic by reflecting the true activities of the AC unit as the author, John Cook, suggests.
3. Can you think of other phenomena that have this oscillatory behavior as a final "settling point?" If so, build a model for your phenomena.

John Cook also brings gems about differential equation in his Twitter feed, Differential Eqns [2]. Check his many grea ideas.

REFERENCES

[1] Cook, John. 2010. *A childhood question about heat*. `https://www.johndcook.com/blog/2010/03/10/cooling/`. Accessed 25 August 2017.

[2] Cook, John. 2017. *Differential Eqns*. `https://twitter.com/diff\_eq`. Accessed 25 August 2017.