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STUDENT VERSION
Investigating Slope Fields

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STATEMENT

In this activity you will use slope fields to explore solutions to various differential equation models. Be sure to include all required equations, generated slope fields, and explanations in your lab report.

1. For each of the following scenarios (a-i),

    (i) write a differential equation to describe the population,

    (ii) generate the slope field for the differential equation,

    (iii) show *at least* three representative solution curves on the slope field,

    (iv) discuss the short-term and long-term behavior of the solutions.

    In your lab report, for each of these scenarios, you should include the differential equation, the slope field showing solution curves (be sure this is on a reasonable window that allows you to understand the solution behavior), and your discussion of the behavior of the solution.

    Be sure the window for your slope field adequately shows the shapes and long-term behavior of the solution curves. A good general starting point for this might be to choose values between 0 and 20 for your independent variable, and between 0 and 50 for your dependent variable, but you might wish to modify these values somewhat for any given problem.

(a) A population grows at an intrinsic rate of 8% per year.

(b) A population grows at a constant rate of 1 unit per year.

(c) A population grows at an intrinsic rate of 8% per year and has immigration of 2 per year.

(d) A population grows at an intrinsic rate of 8% per year and has initial immigration of 1 per year, decreasing linearly at rate of .2 per year.

(e) A population grows at an intrinsic rate of 8% per year and has sinusoidal immigration following the function $7\cos(\frac{\pi}{5}t)$. You may want to play with various viewing windows on this one to be sure you understand the behavior.

(f) A population declines at an intrinsic rate of 8% per year.

(g) A population declines at an intrinsic rate of 8% per year and has immigration of 2 per year.

(h) A population declines at an intrinsic rate of 8% per year and has initial immigration of 2 per year, increasing linearly at rate of 1 per year.

(i) A population declines at an intrinsic rate of 8% per year and has sinusoidal immigration following the function $7\cos(\frac{\pi}{5}t)$. In addition to the usual viewing window, we suggest verifying your thoughts by looking at a window of 0 to 100 for your independent variable, and -15 to 50 for your dependent variable.

2. Plot a slope field for the differential equation $P' = 0.005P(50-P)$. A suggested interval for P is $[0, 100]$.

   (a) Discuss what sort of solution behavior you see for various initial conditions.

   (b) Notice that some initial conditions lead to similar solution behaviors. For each qualitatively different solution behavior you observed in part (a), describe all of the initial conditions which yield similar behavior. (For each type of qualitative behavior, you should give an interval or union of intervals.)

   (c) What meaning might the number 50 have in this differential equation?

3. Plot a slope field for the differential equation $x' = -x + 80 - 40\,\mathrm{sign}(t-6)$. Play with the viewing window until the curves are continuous on the window shown. What could this equation possibly be modeling (you could think in terms of populations, or you could use another type of scenario to describe what you see)? What behavior is caused by the last term of the differential equation, and how does that play into the scenario you describe?

   Note: there is not a typo in sign. The sign function effectively outputs the sign of the argument. That is,

$$\mathrm{sign}(t) = \begin{cases} 1 & \text{if} \quad t > 0 \\ 0 & \text{if} \quad t = 0 \\ -1 & \text{if} \quad t < 0\,. \end{cases}$$