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SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

STUDENT VERSION

MODELING BUBBLES OF BEER

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Abstract: The goal of this project is to set up and numerically solve a first-order nonlinear ordinary differential equation (ODE) system of three equations in three unknowns that models beer bubbles that form at the bottom of a glass and rise to the top. The system solution is then used to verify the model via data collected from a bubble rising in a glass of beer.

Keywords: First-order, nonlinear, system, numerical methods, beer, bubble, Newton's Second Law, Ideal Gas Law, drag force.

Tags: Hadamard, Stokes, sphere, surface area, volume, mole, parameter, Mathematica, Manipulate, MATLAB, M-file.

STATEMENT

1. Read the paper "Quenching a Thirst with Differential Equations" by Martin Ehrismann, provided as *beer.pdf*. [1].
2. Outline Ehrismann's development of a model for bubbles of CO_2 in a glass of beer, including missing details (i.e. write down the mathematics, starting with initial assumptions, show how each equation follows from the previous equation, and end with the system of differential equations (3) on page 416 of [1]).
3. Ehrismann solves the non-linear ODE system (3) numerically and compares the resulting model to measured data. Using a computer algebra system such as Mathematica, Maple, SAGE, etc., solve system (3) numerically and check each of the four cases he discusses:
    a. Bubbles double at top, but rise too fast.
    b. Bubbles rise to top in 3.78 seconds, but don't double.
    c. Bubbles double at top, rise to top in 3.78 seconds, but intermediate values don't match model values.
    d. Model and measured data agree!

    Hand in your work for each part of this question. (The included Mathematica notebook *BeerTestSample.nb*, and pdf version *BeerTestSample.pdf* for those who do not read Mathematica files, is useful for this question.)
4. Are you able to verify Ehrismann's results? Why or why not?
5. Can you find choices for parameters that lead to better agreement between model and data than those found by Ehrismann? Provide work to justify your answer!

REFERENCES

[1] Ehrismann, Martin. 1994. Quenching a Thirst with Differential Equations. *The College Mathematics Journal*. 25(5): 413-418.