1 A simple situation without friction

1.1 Assumptions

• The upward and the right directions are considered as the positive directions.
• The drag coefficient is considered to be zero.
• The frictional force induced by the horizontally blowing wind is considered to be negligible.

1.2 Modeling

1.2.1 Preliminaries

• Referential of the system is \((o, \vec{e}_x, \vec{e}_y)\) with \((ox)\) and \((oy)\) respectively like a horizontal and vertical axis
• Paper describe a parabolic movement it mean that is not uniform
• Summary of the external forces applied on the system are:
  * The weight of paper \(\vec{P}\)
  * The wind speed \(\vec{v}_w\)

• Second law of Newton we have \(\sum \vec{F}_{ext} = m \vec{a}\), where \(m\) denotes the mass of paper, \(\vec{a}\) is the acceleration vector and \(\vec{F}_{ext}\) represents an external force.

1.2.2 Modeling

According to the 2\(^{nd}\) law of Newton, we have \(\vec{a}(t) = \left( \begin{array}{c} a_x(t) \\ a_y(t) \end{array} \right) = \left( \begin{array}{c} \frac{dv_x}{dt} \\ \frac{dv_y}{dt} \end{array} \right) = \left( \begin{array}{c} 0 \\ -g \end{array} \right) \). At initial conditions \(\vec{OP}(t_0) = (x_0, y_0)\), we suppose that

\[
\vec{v}(t_0) = \left( \begin{array}{c} v_x(t_0) \\ y(t_0) \end{array} \right) = \left( \begin{array}{c} \frac{dx_0 \cos \alpha + d_w v_w}{v_0 \sin \alpha} \\ \frac{d_w + d_p}{v_0 \sin \alpha} \end{array} \right).
\]

where \(\alpha\) denotes the angle between the horizontal axis and \(\vec{v}(t_0)\), \(d_w\) and \(d_p\) also are respectively the density of air and the paper. Indeed, we have applied the quantity of movement conservation rule. Thus, by double integration we obtain the equation position of paper given by

\[
\vec{OP}(t) = \left( \begin{array}{c} \frac{dx_0 \cos \alpha + d_w v_w}{d_w + d_p} (t - t_0) + x_0 \\ -\frac{1}{2} g (t - t_0)^2 + (v_0 \sin \alpha) (t - t_0) + y_0 \end{array} \right)
\]

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It follows that \( t - t_0 = \frac{(x-x_0)(1+d_p)}{d_w v_0 \cos \alpha + v_w} \) and
\[
y = -\frac{g}{2(d_p v_0 \cos \alpha + d_w v_w)^2} \frac{(d_w + d_p)(x - x_0)}{d_p v_0 \cos \alpha + d_w v_w} + y_0
\]
where \( v_0 \) is the initial speed of paper(s), \( y_0 \) is the initial position of paper(s) on the vertical, and \( y \) is the final position of the paper.

2 C**onsidering the frictions of air**

2.1 Assumptions

- Referential of the system is \((\alpha, e_x, e_y)\) with \((\alpha x)\) and \((\alpha y)\) respectively like a horizontal and vertical axis.
- Material (paper) describes a parabolic movement meaning that it is not uniform.
- Summary of the external forces applied on the system are:
  - * The weight of paper \( \vec{P} \)
  - * The wind speed \( \vec{v} \)
  - * The friction of air \( f_w \)
- Second law of Newton we have \( \sum \vec{F}_{ext} = m \vec{a} \)

2.2 Modeling

According to the 2\(^{nd}\) law of Newton, we have \( \vec{a} \left( \begin{array}{c} a_x(t) \\ a_y(t) \end{array} \right) = \left( \begin{array}{c} \frac{d}{dt} a_x(t) \\ \frac{d}{dt} a_y(t) \end{array} \right) = \left( \begin{array}{c} -f_w v_x \\ g \end{array} \right) \) At initial conditions we suppose that \( \vec{OP} = \left( \begin{array}{c} x(t_0) \\ y(t_0) \end{array} \right) = \left( \begin{array}{c} x_0 \\ y_0 \end{array} \right) \) and \( \vec{v}(t_0) = \left( \begin{array}{c} \frac{dx(t_0)}{dt} \\ \frac{dy(t_0)}{dt} \end{array} \right) = \left( \begin{array}{c} d_p v_0 \cos \alpha + d_w v_w \\ v_0 \sin \alpha \end{array} \right) \). Thus, we have according to we have
\[
\frac{dv_x}{dt} = -\frac{f_w}{m} v_x \] which implies
\[
v_x = \frac{d_p v_0 \cos \alpha + d_w v_w}{d_p + d_w} e^{-\frac{f_w}{m} (t-t_0)}. \]
and
\[
v_y = \left( \frac{m}{f_w} g + v_0 \sin \alpha \right) e^{-\frac{f_w}{m} (t-t_0)} - \frac{m}{f_w} g. \] (3)

By integrating the above expressions
\[
\vec{OP} = \left( \begin{array}{c} \frac{m(d_p v_0 \cos \alpha + d_w v_w)}{f_w d_w + d_p} \left( 1 - e^{-\frac{f_w}{m} (t-t_0)} \right) \\ y_0 + \frac{m}{f_w} \left( \frac{m}{f_w} g + v_0 \sin \alpha \right) \left( 1 - e^{-\frac{f_w}{m} (t-t_0)} \right) - \frac{m}{f_w} g (t-t_0) \end{array} \right). \] (4)

Notice that it is necessary to have
\[
x_f < \frac{md_w}{k \eta (d_w + d_p)} \quad \text{or} \quad v_0 > \left( \frac{k \eta f_m}{m \cos \alpha} \left( \frac{d_w}{d_p} + 1 \right) - \frac{d_w}{d_p \cos \alpha} \right) v_w \] (5)

3 Discussion

During this work, we have considered two situations. In the first case friction forces are neglected, while they are considered in the last case. We stated differential equations governing the system and solved them both analytically and numerically. The impact of density of materials was considered and appeared into simulations (available in a .gif picture). The choice of the wind speed \( v_w \) and the settler diameter \( x_f \) involves a minimal height depending on physical properties of the system.

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