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Problem C: Modeling the Cool Kids
Definitions of Groups

• The American Heritage Dictionary definition of a group:
  ▪ A number of individuals or things considered or classed together because of similarities

• The Merriam-Webster Dictionary definition of a group:
  ▪ A number of individuals assembled together or having some unifying relationship

• Our definition of a group:
  ▪ A socially based cohesive assemblage of individual students.
Our Approach

- We wanted a model that was able to account for many groups and that included a carrying capacity. Carrying capacity is represented in our model as the most stable number of groups a population can hold.

- We wanted a model that agreed with the results from the study on homophily in academic performance, which proposed three relevant assumptions:
  - GPA and age are the driving factors behind initial formation of groups.
  - Students will change groups rather than their educational performance.
  - Students’ GPAs remain relatively constant over the timeline of their educational experience.

- From these considerations, a logistic growth model was chosen.
Our Assumptions

• After the initial formation of groups, most students will stick with their first chosen group.

• All students are interested in being members of a group.
  - We considered the number of individuals uninterested in being in groups to be less than 1% of any given total population and therefore do not significantly affect our logistic analysis of the groups.

• There will never be fewer than one group in a population, as the total population itself can be considered a group.

• The average stable group size is 5 students. We assumed this value based on our own personal experience as college and high school students.
Our Model

Equation 1: Group Growth Model

\[
\frac{dG}{dt} = kG \left(1 - \frac{G}{E}\right)
\]

- \( G \) is the total number of groups formed as a function of time.
- \( t \) is time, measured in months.
- \( E \) is the number of groups that a given population can stably hold. From our experiences, the most stable group size was estimated to be five students. Therefore, \( E \) can be expressed as
  \[ E = \frac{p}{5} \]
- \( k \) is a constant representing the natural growth rate of groups, and is expressed in units of inverse months.
Graph of Our Model
The equilibrium solutions of the model are where $G = E$, and where $G = 0$.

- When $G$ is 0, the population must be zero, so the rate at which groups form will remain constant at 0.
- If $G = E$, the system will be stable, as the total number of groups will be equal to that supported by the population.

\[
\frac{dG}{dt} = kG \left(1 - \frac{G}{E}\right)
\]
What Happens Outside Equilibrium?

\[ \frac{dG}{dt} = kG \left(1 - \frac{G}{E}\right) \]

- When \( G < E \), the fraction \( G/E \) is less than one, and the rate of change of \( G(t) \) is positive. Conceptually, this is consistent with what is expected: when there is a smaller number of groups in a population than it can hold, groups will break apart, approaching an average of five students per group.

- When \( G > E \), the fraction \( G/E \) is greater than one, therefore the rate of change of \( G(t) \) is negative. Again, this is consistent with what is expected: when there is a larger number of groups in a population than it can hold, groups will merge together, approaching an average of five students per group.
As Equation 1 is a logistic growth model, it can be solved to isolate $G(t)$, as shown in Equation 2. If the initial number of groups, total population, and group growth rate constant $k$ are known, the number of groups at a given time can be calculated.

**Equation 2: Defining $G(t)$**

$$G(t) = \frac{G_0E}{(E-G_0)e^{-kt}+G_0}$$

In many cases, $G_0$ is greater than one. Assuming that $t = 0$ represents the start of a school year, there may already be pre-existing groups in the population at that time.

If $G_0$ is small, then the initial $dG/dt$ values will be small; after a short time period, they will increase dramatically. As the number of groups approaches equilibrium, the rate will approach zero asymptotically.

Reasonable values of $k$ are between $k=1$ and $k=4$. 

What We Can Determine
Our model describes a dynamic equilibrium: It accounts for minor GPA fluctuations by averaging them out over the whole population. Our model does not account for if students had frequent and dramatic changes in GPA; however, from studies and our experience as students, this does not often occur.

While our model provides an overview of the rates of group formation, it does not model the interactions between groups, instead averaging these into our dynamic equilibrium.

Our model is only applicable to measuring group formation when $t=0$ represents the start of a school year or term.

Our model does not account for students uninterested in joining groups, as it is assumed that they are too small of a percentage of the population to greatly affect the growth rate.

When the number of groups is increasing, the last five or fewer students in a population will never form a group because $G(t)$ never reaches $E$. Similarly, when the number of groups is decreasing, there will always be two undersized groups.

This model will only work for a student population of 5 or greater. For $P < 5$, the value of $G$ will decrease over time.
Further Implications of Our Model

\[ \frac{dG}{dt} = kG \left(1 - \frac{G}{E}\right) \]

- Once the growth rate nears equilibrium, the number of groups stays roughly the same. This agrees with what was found in the study on the importance of GPA in group formation.
- The equilibrium is based on an average, so that if minor GPA fluctuations do occur for a student, they will be averaged out over the population. This can be used to explain the strength of most high school and college groups that are linked by similar GPAs, as shown in the next slide.
Rich Kids: Medium-High GPA

Nerds: High GPA
(As long as they’re not playing too many videogames)

Socies: Medium-High GPA

Stonies: Low GPA

Outsiders: High GPA

Jocks: Medium-Low GPA

The Molly Ringwalds: Medium-High GPA
Our model does not account for group interactions, as it is based on a dynamic equilibrium where only minor fluctuations can occur. Because of this, the additional issue cannot be included into our current group formation model.

We have instead proposed an additional set of equations which incorporate elements of a logistic growth model and a competing species model.

Our new model is based on the assumption that groups can contain members of different political parties, and it analyzes the rates at which these different parties influence each other.
Additional Issue

Additional issue equations

\[
\begin{align*}
\frac{dR}{dt} &= \alpha R \left( 1 - \frac{R}{P} \right) - c(RD + RN) \\
\frac{dD}{dt} &= \beta D \left( 1 - \frac{D}{P} \right) - c(DR + DN) \\
\frac{dN}{dt} &= \gamma N \left( 1 - \frac{N}{P} \right) - c(NR + ND)
\end{align*}
\]
\[
\frac{dR}{dt} = \alpha R \left(1 - \frac{R}{P}\right) - c(RD + RN)
\]
\[
\frac{dD}{dt} = \beta D \left(1 - \frac{D}{P}\right) - c(DR + DN)
\]
\[
\frac{dN}{dt} = \gamma N \left(1 - \frac{N}{P}\right) - c(NR + ND)
\]

- \(R\) represents the total republican population, \(D\) the total democrat population, and \(N\) the total unaffiliated population. The total population is defined as the sum of these.
- \(\alpha, \beta,\) and \(\gamma\) represent the natural growth rate coefficients. These represent the rate at which the populations of these parties would grow if they were isolated from the influence of the other parties, and if there was no carrying capacity.
- We included carrying capacities for each party, which limit the possible number of party members to the total population.
- It is assumed that each party has equal influence on all other parties, therefore making it so that no one party is disproportionately superior over the others.
- The last term of each of the equations describes the interactions between the political parties.
The End