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An important step on recycling deals with sorting paper and cardboard, which are materials difficult to separate.

The goal is to design a process that will allow for the separation of 30-40% of the paper from the cardboard by dropping them from a height and blowing them with a fan.
The fan used is one manufactured by Impact Air Systems. For this model, we used the chopper fan model 30 with dimensions $3.60 \text{m} \times 2.15 \text{m}$.

We placed the top of this fan at half the total height $h$ which the items are going to travel.

This model assumes air resistance to be negligible in all dimensions.

We assume the cardboard and paper to have relatively uniform distributions.
Density of Materials

The cardboard and paper have relatively continuous uniform distributions; samples of each category are listed below

**Paper Density**
- Newspaper: 48.4 \( g/m^2 \)
- Paper: 75 \( g/m^2 \)
- Cardstock: 300 \( g/m^2 \)

**Cardboard Density**
- Cereal Box: 385 \( g/m^2 \)
- Single Corrugated Cardboard: 450 \( g/m^2 \)
- Triple Corrugated Cardboard: 1370 \( g/m^2 \)

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1 Measured empirically
Modeling the motion of recyclable items

Before fan

\[ \sum F_x = 0 \]
\[ \sum F_y = -mg \]

At fan

\[ \sum F_x = T_h = m\ddot{x} \]
\[ \sum F_y = -mg \]

After fan

\[ \sum F_x = 0 \]
\[ \sum F_y = -mg \]
Thrust provided by fan

\[ T_h = \frac{1}{2} c_w \rho A v_{\text{wind}}^2 \]

Let \( \alpha = \frac{1}{2} c_w \rho A \)

Time spent at the fan is dependent on \( h \)

\[ \Delta t_{\text{fan}} = \frac{\sqrt{h+7.20} - \sqrt{h}}{\sqrt{g}} \]

Motion in \( y \) is given by

\[ \ddot{y} = -mg \]

Motion in \( x \) is given by

\[ \dot{x} = \frac{\alpha}{m} v_{\text{wind}}^2 \]
The final displacement of the paper/cardboard depends on the height at which they were released, the mass of each item, and the wind speed.

\[ x = \frac{\alpha v_{\text{wind}}^2}{m} \left[ \frac{\sqrt{2h^2 + 14.40h} - (h + 7.20) - h\sqrt{2 + \sqrt{h^2 + 7.20h}}}{g} \right] \]
Estimating the Motion of different recyclable items

\[ x = \frac{\alpha v^2_{\text{wind}}}{m} \left[ \frac{\sqrt{2h^2+14.40h-(h+7.20)}-h\sqrt{2}+\sqrt{h^2+7.20h}}{g} \right] \]

- Through iterations of a code, we minimized the height and wind speed for which the displacement of the lightest cardboard piece was at least 1.0 m. (Decided not to have the fan on the ground and considered heights greater than 7.51 m in our code.)

- Next, we established a "distinguishable" separation between the cardboard and paper bins: 0.5 m.

- After setting this separation, we could determine the amount of paper that at this height and wind speed would be displaced an "indistinguishable" distance, and would fall into the cardboard region.
We found the minimum height to be $h = 7.55$ m and the minimum wind speed to be $v_{wind} = 9.89$ m/s.

Under these conditions, all paper under the density of 256.74 $g/m^2$ will fall in the paper bin. Thus, heavier paper (e.g. cardstock will fall in the cardboard bin).

This corresponds to about 82% of the paper being properly separated (not including crumpled or deformed items).

The bins used to sort the materials should be 1.5 m for the cardboard and 6.5 m for the paper.
What aspect has the largest effect in sorting quality given a small change?

- The sorting quality of this model is dependent on the height $h$ and fan speed $v_{\text{wind}}$.
- Studied this by setting one parameter constant and varying the other in steps of 0.1.
- The sorting quality is proportional to the displacement of the lightest cardboard ($x_{CB}$), we generated plots of:
  
  $x_{CB} \text{ vs. } h$
  
  $x_{CB} \text{ vs. } v_{\text{wind}}$
A small change in height results in a larger difference in sorting quality than a small change in velocity.
References
