Separating Recyclables

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April 21, 2018

1 Introduction

We focused on the problem of separating recyclable materials, specifically how to separate paper from cardboard. To do this we looked at a column of falling paper and cardboard that had a fan blowing across it at 90 degrees. Our problem was to determine the optimal height of the column and the force of the wind blowing across it to get the majority of the paper out so that only cardboard fell to the bottom.

For this model we consider a few different factors: the shape and mass of the materials, the orientation of the materials and the angle of the fans. While discussing these factors we began making assumptions about the materials and generated the following list:

- the boxes have been crushed flat
- all materials have been through a cutting process and are roughly the size of a standard sheet of paper
- due to its thickness, cardboard will weigh more than paper
- the number of cardboard and paper pieces falling is about even
- the papers are in no way attached to each other

Using these assumptions we moved on to build a model of the falling objects.

2 Model

We began with the net force equation and the broke it into its $x$ and $y$ components to get:

\[ F_{\text{net } x} = F_{\text{fan}} - F_{\text{drag } x} \]
\[ F_{\text{net } y} = F_{\text{drag } y} - F_g \]

\[ F_{\text{drag}} = \frac{1}{2} A \mu \rho v^2 \] where $\mu = 1.05$ a drag constant, $\rho = 1.1644 \text{kg/m}^3$ the density of air at $30^\circ\text{C}$, and $A$ is the cross sectional area of the object. Since paper and cardboard twist and turn as the fall we assumed an average position of $45^\circ$ and found an area $= 0.042664 m^2$.

Using these equations we ran a numerical integration to find the distance traveled in both directions along with the time taken. During the fall the objects had a 25% chance of hitting something, during which time they did not move in the $x$ direction. The formulas used are given in the chart.
below, along with the update conditions.

<table>
<thead>
<tr>
<th>time</th>
<th>$a_y$</th>
<th>$v_y$</th>
<th>$y$</th>
<th>$a_x$</th>
<th>$v_x$</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$-9.8$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$+\Delta t$</td>
<td>$\frac{1}{m}(0.0261v_y^2) - g$</td>
<td>$v_y + a_y\Delta t$</td>
<td>$y + v_y\Delta t$</td>
<td>$F_{fan}$</td>
<td>$v_x + a_x\Delta t$</td>
<td>$x + v_x\Delta t$</td>
</tr>
</tbody>
</table>

Ultimately $\Delta t$ was used as 0.005 in order to get an accuracy better than 0.5%. We ran multiple simulations varying both the height and force required and found that 2 m in fall height and a force of 0.08 N were enough to get almost 2 m of separation between the paper and cardboard.

### 3 Conclusion

Using the force we found, we discover that we would need a fan capable of moving 169.51 cfm (cubic feet per minute) at a speed of 1 m/s to get the amount of force required. There would also need to be a 2 m wall of these fans on one side in order to get constant force across the column. Given that fans capable of this are available we concluded that this would be a viable way to separate paper and cardboard.