Alarm Bells

We were asked to model a prey’s response to an approaching predator. The model is expected to use simple concepts like distance and speed in order to predict complex decisions.

Model:

The model assumes that the prey’s response—measured by the rate at which it expends energy—depends on the amount of energy it possesses at any given moment; the separation between predator and prey at that moment; and the rate at which this separation is changing (Bhattaacharyya et al., 2017, p.2751).

\[ \frac{dE}{dt} = E \cdot \frac{d(separation)}{dt} \cdot \frac{1}{separation} \]

Here, \( \frac{dE}{dt} \) is always negative because the prey is only ever going to expend energy; \( E \) is always positive; distance is always positive.

The model assumes that the prey’s speed of recession, \( v(t) \), is proportional to the rate at which it expends energy. The model also assumes that both the prey and predator only travel in straight paths. The negative sign ensures that the speed is positive since \( \frac{dE}{dt} \) is always negative.

\[ V(t) = -\frac{dE}{dt} \]

The separation between predator and prey at any given moment, \( r(t) \), is equal to the distance from the predator to the starting position of the prey, \( R(t) \), plus the distance that the prey has moved up until that moment. We choose \( R(t) \) to be a function that starts positively and decreases because a predator will start a finite distance away from a prey and approach the prey’s location. Also, since the prey reacts only when the predator is within a critical distance, \( R(0) \) may be defined as \( d_{crit} \) because the predator has to start farther away than this critical distance and the prey is triggered when the predator arrives at this point. In this way we simplify our differential equation while leaving the results unchanged.

\[ r(t) = R(t) + \int_0^t V(x) \, dx = R(t) + E(0) - E(t) \]

Consequently, the rate of change of separation at any given moment, \( r'(t) \), is dependent on the following manner upon the rate at which the predator’s position is changing relative to the prey’s starting position, \( R'(t) \), and the rate at which the prey expends energy:

\[ r'(t) = R'(t) - \frac{dE}{dt} \]

Combining all of the above assumptions and deductions, we arrive at the following differential equation:
\[
\frac{dE}{dt} = E \cdot \frac{R'(t)}{E(0) + R(t)}
\]

This tells us that the rate at which the prey evades at any given moment is directly proportional to the rate at which the predator is chasing the prey; the energy of the prey at that moment; and the distance at that moment of the predator from the prey’s starting position.

We can solve the above equation using the method of separation of variables to get the following equation for the energy of the prey at any given moment:

\[
E = \left( \frac{E(0)}{E(0) + R(0)} \right) \left( R(t) + E(0) \right)
\]

If we suppose the predator only chases the prey until a time \( p \), at which point the fish has successfully evaded capture, and repeat this process multiple times with the same attack approach, we can iteratively define the fish’s energy after the \( i \)-th attack by

\[
E_0 = E(0) \quad E_i = \frac{E_{i-1}}{E_{i-1} + R(0)} \left( R(p) + E_{i-1} \right)
\]

In the graph, the red and black lines are \( R(t) \) and \( E(t) \), respectively. The blue line is the distance traversed by the zebra fish at time \( t \). Here, the zebrafish and predator both settle to an equilibrium, i.e. both of them come to rest.

References: