Statement of Problem: Design a differential equation model to determine whether an animal flees a predator due to the size and rate of change of size of the predator.

We will be testing our solution with an example of the Arctic Hare from the Arctic tundra. The largest predators are grey wolves, with an average frontal area of $a = 0.418064 \, \text{m}^2$; the smallest predators are arctic foxes, with an average frontal area of $a = 0.0464515 \, \text{m}^2$. The fastest predators have an average rate of change of size of $4.3137 \, \text{m}^2/\text{s}$, and the slowest have an average rate of change of size of $3.0159 \, \text{m}^2/\text{s}$. We will be testing our model on a Grey wolf attacking an Arctic hare from a distance of 100 m. We assume that from the prey’s point of view, the predator is circular in shape. The speed of the predator with respect to time is constant in our model. The speed of the predator and the size of the predator both affect the prey with the same magnitude.

The first formula we found was the formula for the frontal area of the predator, from the perspective of the prey, with respect to its distance from the prey.

$$A = \pi y^2 = \pi (k(100 - x) + r)^2$$

$A$ is the frontal area of the predator; $y$ is the radius of the predator at any distance, and is given by the formula $y = k(100-x) + r$, where $k$ is a constant, $r$ is the initial radius of the predator at a distance of 100 m, and $x$ is distance from the prey. We assumed that from the perspective of the prey, the predator is circular in shape. This allowed us to measure frontal area using the formula for the area of a circle. We used a linear function to model $y$, since we assumed the speed is constant, and therefore the radius of the animal would increase at a linear rate. This equation is used to find the frontal area of the predator in terms of the distance between the predator and prey. We can also use this formula to find the rate of change of area with respect to time, or the rate of change of size, without using a specific radius.

$$\frac{dA}{dt} = 2\pi (100k - kx + r)(-k \times \frac{dx}{dt})$$

$rac{dA}{dt}$ is the rate of change of the frontal area of the predator with respect to time, and $\frac{dx}{dt}$ is the rate of change of distance with respect to time, which is velocity. Since we assumed that the speed of the predator is constant throughout the 100m, $\frac{dx}{dt}$ is simply the constant speed of the predator.

$$\frac{dx}{dt} = \frac{A'}{2\pi(100k - kx + r)}$$

$\frac{dx}{dt}$ is the velocity of the predator with respect to time, and $A'$ is the rate of change of area with respect to time, which was found above. In the grey wolf example, the velocity is 17.88 m/s. Our next step was to find out whether or not the prey will flee. We used an equation with two variables, $F$. This allowed us to find whether or not the animal will flee, dependent on the value of $F$.

$$F(a, v) = A + V = A_0(10/(a_{\text{max}} - a_{\text{min}})) + V (10/(v_{\text{max}} - v_{\text{min}}))$$

$A$ is the size of the predator; $V$ is the velocity of the predator; $a_{\text{max}}$, $a_{\text{min}}$, $v_{\text{max}}$ and $v_{\text{min}}$ are constants depending on the predator. In this example $a_{\text{max}}$ is 0.418064 m$^2$, $a_{\text{min}}$ is 0.0464515 m$^2$. 
\( v_{\text{max}} \) is 17.88 m/s, and \( v_{\text{min}} \) is 13.412 m/s. \( F \) is the function that will determine whether the prey flees or not. If the resultant value of \( F \) is greater than or equal to 10, the animal will flee. Otherwise, it will not. There are two parts to this equation: size and velocity. We assumed that they are both equally weighted in the prey’s decision. The size of the predator and its velocity are scaled to numbers between 0 and 10. If the sum of these numbers is greater than or equal to 10, we believe the prey will flee.

This graph shows the size of the predator in relation to the prey. As the predator approaches the prey, its frontal area increases. The radius of the assumed circular frontal area increases linearly as distance decreases. The distance between predator and prey is initially assumed to be 100m. On this graph, the prey is at the y-axis. The y-axis represents height of the predator, which shows that the radius and frontal area both increase as the predator approaches. This graph allows us to calculate the speed of the predator, as we can see the correlation between frontal area, radius, and distance clearly.