Collisions are minimal - allows for the use of continuous kinematics.
Correctional Constant - as C approaches a maximum, the model will account for the slowest possible correction of orientation.
Density of cardboard - corresponding to Amazon box test article.
Density of paper - corresponding to standard office paper.
Kinematics - Initial velocity components are zero and initial x and y positions are zero and height, respectively
Aerodynamics - local velocity varies linearly with paper/cardboard orientation; paper will not flex; fan flow static pressure is equal to air pressure in the chute
Progression of models

1) Particle physics with fan forces and acceleration of gravity
2) Include random orientation of paper to observe effects of fan
3) Incorporate air resistance into the paper and the cardboard models
4) Maximize correctional constant to observe worst case scenario
5) Analysis of most sensitive variable (fan velocity)
Correctional Constant

We assume that an object influenced by the fan will eventually reach an orientation orthogonal to the flow of the fan speed. We assume this because the flow is constant throughout the entire drop area, therefore the object is forced into an orthogonal orientation. The correctional constant $C$, forces the model to adjust the orientation of an object over time until it reaches an orientation orthogonal to the flow.

$C$ is equal to the maximum time divided by the step in time when the trajectory is computed. The max time is equal to 1.0s. The time step is equal to .001s.

Although constant in our model, the value of $C$ may be modified in order to incorporate variable rates of orientational correction. The bounds for $C$ are as follows:

$$C \in [1, T]$$

$$T = \frac{T_{\text{max}}}{T_{\text{step}}}$$

$$C = \frac{1}{C} \times \phi_i$$

$$\phi_{i+1} = \phi_i - \frac{1}{C} \times \phi_i$$

$$T_{\text{max}} = 1.0$$

$$T_{\text{step}} = .001$$
Schematic of Aerodynamic Forces

\( q_{\infty F,O} \) “dynamic pressure of orthogonal component of fan stream”

\( q_{\infty P} \) “dynamic pressure of flow opposite plate velocity \( V \)”

\( C_D = 1.28 \) “drag coefficient for an orthogonal plate” [2]

\( \overline{V_F} \) “fan speed”

\( \rho \) “air density”

\( \phi \) “cardboard/paper angle”

\[ \sum \overline{F_x} = \overline{R_F \cos(\phi)} - \overline{D \cos(\phi)} \]

\[ \sum \overline{F_y} = \overline{R_F \sin(\phi)} + \overline{D \sin(\phi)} \]

\( \overline{R_F} = dP \overline{A} \) for \( dP = C_D q_{\infty F,O} \)

\[ q_{\infty F,O} = \frac{1}{2} \rho \left[ \overline{V_F \cos(\phi)} \right]^2 \]

\( \overline{D} = C_D q_{\infty P} \overline{A} \) for \( q_{\infty P} = \frac{1}{2} \rho V^2 \)
Final Model Outputs

Approximated trajectories of paper(left) and cardboard(right) where fan speed is 1m/s and drop height is 1m.
Approximated trajectories of paper(left) and cardboard(right) where fan speed is 2m/s and drop height is 1m.
Approximated trajectories of paper(left) and cardboard(right) where fan speed is 3m/s and drop height is 1m.
Approximated trajectories of paper(left) and cardboard(right) where fan speed is 1m/s and drop height is 0.5m. (Optimal solution)
Approximated trajectories of paper(left) and cardboard(right) where fan speed is 2m/s and drop height is 0.5m.
Approximated trajectories of paper(left) and cardboard(right) where fan speed is 3m/s and drop height is 0.5m.
Additional Issue

1. Which aspects of your model results in the largest difference in sorting quality if that aspect undergoes a small change?
2. A small change in velocity results in a more significant change in sorting quality than changing the drop height.
3. Based off of various configurations, the boundary of separation may be estimated by a function of velocity and height, or $x = f(v, h) = \frac{1}{2}vh$. 
Reflections

For a more accurate model

➔ Correctional constant = f(van velocity) therefore no longer a constant
➔ Inclusion of viscous drag for flat falling cardboard and bendable paper
➔ Adaptation of Kutta-Joukowski Theorem to estimate spinning object trajectory

Error sources in current model

➔ Uniform size of paper and cardboard is not likely
➔ Equal mass distribution does not perfectly reflect 2013 recycling recover rates
➔ Orthogonality condition (correctional constant) does not represent the probable lowest energy state
\[ F = P_0(-\bar{A}) \]
\[ F = 0.9 \text{ atm}(100 \text{ cm}^2) \]
\[ \bar{V} = \frac{N}{\text{cm}^3} \text{ (volume)} \]

\[ V_{\text{water}} = 916 \text{ mm}^3 \]
\[ m = 0.7474 \text{ g} \]
\[ d_e = dQ + dW \]
\[ dW = \frac{F \cdot dP}{\partial P} \]

\[ P_{\text{max}}(\bar{A}) = \int_{\text{cs}}^{\text{cs}} \frac{dV}{V} \left( \rho_v \cdot dA \right) \]

\[ P = f(v) \]
\[ V = f(w) \]

\[ A = 100 \text{ cm}^2 \]
\[ \rho = 1.225 \times 10^3 \text{ g/mm}^3 \]
\[ V_p = 3.000 \text{ mm/s} \]
\[ M = 0.7474 \text{ g} \]
1. \( \bar{F} = \bar{p} \cdot \chi f(V, \phi) \)  
   \[ \bar{a} = f(v, \phi) = \left( \frac{P_v}{P_{\text{max}}} \right) \bar{p} \]  
   \[ \frac{v}{\text{Mcpl}} \]  
   \[ (0-2)(P_{\text{max}} \bar{A}) \]  
   \[ \text{Mcpl} \]  
   \[ - \]  
   \[ \bar{V}_p = \bar{a} t_{0 \rightarrow AR} \]  
   \[ \bar{V}_{p2} = \bar{V}_p \text{ (no occ. by far after } t = AR) \]  

2. \( y(t) = y_0 + V_{F2y} t - V_{y0} t - \frac{1}{2} a_2 t^2 \)  
   \[ x(t) = x_0 + V_{F2x} t + V_{x0} t \]  
   \[ N(V, \theta, h) = \% \text{ of } p_{\text{ap}} \]  
   \[ \text{Model Set-Up} \]

Paper:
\[ A = 100 \text{cm}^2 \]  
\[ \rho = 1.225 \times 10^3 \text{ g/cm}^3 \]  
\[ V_p = 3000 \text{mm/s} \]  
\[ M = 0.2 \]  
\[ \text{Temperature of Pan Fluid} \]  
\[ L \]  
\[ \bar{F} \]  
\[ \bar{A} \]  
\[ \bar{V} \]  
\[ \bar{p} \]  
\[ \chi \]  
\[ f(V, \phi) \]  
\[ P_{\text{max}} \]  
\[ P_v \]  
\[ v \]
\[ p = \frac{\rho v^2}{g} \]

Unit volume:

\[ \rho V_{EA} A_s \]

\[ P_{EA} = P_{EA} A_s + [x p_{EA}] V_{EA} A_s \]

\[ x = \cos \theta \]

\[ y = r \sin \theta \]

\[ \theta = \frac{\pi}{4} \]

\[ \theta = \frac{1}{2} \pi \]

\[ \begin{align*}
  \rho v^2 & = \frac{p}{\rho} \\
  P_{EA} & = p_{EA} A_s + [x p_{EA}] V_{EA} A_s \\
  P_s & = p_s - p \end{align*} \]

\[ P_s = P_0 - z \]

\[ P_0 = 11.025 \text{ Pa} \]

\[ \rho \approx 5.5125 \text{ Pa} \]

\[ \begin{align*}
  \text{orient} & [1 \ 0 \ 1] \\
  \text{force}_x & [0 \ \rho A \cos 0] \\
  \text{force}_y & [0 \ \rho A \sin 0] \\
  V_{local} & \begin{bmatrix} v_x & 0.5v_x & 0 \end{bmatrix} \\
  p & \left[ \begin{bmatrix} \rho - 2\rho \end{bmatrix} \right] \\
  S(v) & \left[ \begin{bmatrix} p_0 \end{bmatrix} \right] \end{align*} \]
\[ \rho_c = \rho_p \rightarrow \text{equal distribution} \]

\[ \frac{m_c}{V_c} \]  
\[ \frac{m_p}{V_p} = y = \rho_p \]

\[ \frac{\rho_c}{\rho_p} = 1 = \frac{m_p}{V_p} \]

\[ \sum m_c = 1 \]

\[ \sum V_c = \sum V_p \]

\[ \frac{\sum V_c}{\sum V_p} = 1 \]

\[ \frac{m_c}{m_p} (\text{in system}) \]

\[ \rho_c = \rho_p \left[ \frac{\text{cm}^3}{m^3} \right] \rho_c = 1 \]

\[ \sum m_c = \sum m_p \]

\[ \frac{\sum m_c}{\sum V_c} = \frac{\sum m_p}{\sum V_p} \]

\[ \text{ratio of volume: Card: Paper} \]
orient $[0 \ \frac{\pi}{4} \ \frac{3\pi}{2}]$

force $x$ $[0 \ \frac{\vec{PA}}{m_{\text{age}}} \ \frac{\vec{PA}}{m}]$

force $y$ $[0 \ -\frac{\vec{PA}}{m_{\text{age}}} \ 0]$

$V_{\text{local}}$ $[V_k \ 0.5V_k \ 0]$

$P$ $[P_0 - 2\tau \ s(V^2) \ P_0]$
Final forces:
- Paper
  1) Orthogonal to spin
  2) Spin → lift (bd)
  3) Precession
  4) Cardboard

Constants:
- $C_D = 1.28$
- $C_D = 1.56$
- Drag = $C_D \frac{1}{2} \rho V^2 A$

Equations:
- $\dot{\phi} = -\frac{1}{2} \dot{\theta}$
- $\phi < 0 \rightarrow P = -\hat{b}_y$
- $\phi > 0 \rightarrow P = \hat{b}_y$
- $\phi = 0 \rightarrow P = 0$
- $\phi : 0 < \pi \rightarrow \dot{\theta} = \frac{1}{2} \dot{\phi}$
- $\phi : 0 > \pi \rightarrow \dot{\theta} = -\frac{1}{2} \dot{\phi}$
- $\phi : 0 = \pi \rightarrow \dot{\theta} = 0$
- $\phi : 0 > \frac{3\pi}{2} \rightarrow \dot{\theta} = 0$
- $\phi : 0 < \frac{3\pi}{2} \rightarrow \dot{\theta} = \frac{1}{2} \dot{\phi}$
- $\phi : 0 = \pi \rightarrow \dot{\theta} = 0$
- $\phi : 0 > \pi \rightarrow \dot{\theta} = -\frac{1}{2} \dot{\phi}$
- $\phi : 0 < \pi \rightarrow \dot{\theta} = \frac{1}{2} \dot{\phi}$
- $\phi : 0 = \pi \rightarrow \dot{\theta} = 0$
- $\phi : 0 > \frac{3\pi}{2} \rightarrow \dot{\theta} = 0$
- $\phi : 0 < \frac{3\pi}{2} \rightarrow \dot{\theta} = \frac{1}{2} \dot{\phi}$
Steps to produce final model:

1) Particle physics with initial velocity of fan and gravity.
2) Randomness of orientation while moving through air.
3) Incorporated air resistance (drag) into model.

\[ \alpha_x = \frac{-\Delta F_x}{m} \]
\[ \alpha_y = \frac{-\Delta F_y}{m} \]

\[ \Sigma F_x = D_f + 2 \epsilon \gamma \Delta \Psi \]
\[ \Sigma F_y = D_{up} + \rho V^2 \Delta \Psi \]
\[ Re_c = \frac{R \Psi_{11}}{x} \]

Paper: \[ \Sigma F_x = \int \rho A s \cos \theta - \rho A \Delta x \sin \theta \]
Given: $P_0$ as $pV_f^2$

$P_e = \frac{1}{2} pV_f^2 + P$

$P_n = \frac{1}{2} p(3)^2 + 101325$  

\[ \begin{bmatrix} -\frac{V_f}{2} & 0 & \frac{V_f}{2} \end{bmatrix} \text{ steps} \]

\[ \begin{bmatrix} 0 & V_f & 0 \end{bmatrix} \text{ linear and equal steps} \]

\[ \int \frac{dp}{\rho} \begin{bmatrix} 0 & 0 & 0 \\ C_{D200} & C_{D200} & C_{D200} \end{bmatrix} \]

$F = dpA$

$P_c = 101325 \text{ N/m}^2$

$P_e = P_c$

$P_n = P_e$

$P_n - P_c = 11.9 = \frac{1}{2} pV_f^2$

$101325 \times 5 \times \frac{1}{2} (1.25)^2 \times (3)^2$

$= 6.28 \times 101325 \times 1.25$

$= 7.056$

$C_D = \text{Flat Plate}$

$R = C_D p_{200} A$

$\bar{R} = C_D p_{200} V_f A$

$D = C_D p_{200} A$

$C_D = 1.28$

$V_{f0} = V_f \cos \alpha$
**Paper**

\[ \Sigma F_x = F_x \cos \theta - D \cos \phi \]
\[ \Sigma F_y = F_y \sin \phi - W \]

**Cardboard**

\[ \Sigma F_x = D \]
\[ \Sigma F_y = -W \]

**Rejection**

\[ \frac{\rho V F E}{\mu} \]

\[ R = C D 2 \mu \rho A \]
\[ D = C D 2 \mu \rho A \]

\[ C_D = 1.28 \]

**Replacements**

\[ F_x = d P A \]
\[ 2 \rho \nu = \frac{1}{2} \rho \left( V_x \cos \phi \right)^2 \]

\[ C_D = 1.28 \] "flat plate"
Libraries

Code written in Python

- Matplotlib: Plotting data
- NumPy: Scientific computing

```python
import numpy as np
import matplotlib.pyplot as plt
```
References

