Modeling for Cool Kids

I. Introduction

For this problem, we were asked to model students in high school or college as they transition between different social groups, also known as cliques, using the fewest number of variables possible. We did this by adapting a model used to represent the transition of voters between political parties prior to a presidential election [1].

II. The Model

We based our model on a simplified version of a system of differential equations used to show the rates of change of political parties prior to an election. After examining the model presented in the paper, we used the concept of Markov Chains to visualize how we should modify the equations to suit our problem. We modeled this interaction for two and three groups and theorized how we could generalize our equations to work for more groups.

After reading the problem, we made assumptions to help develop the model.

- The population of the system is constant.
- Time will be measured in months over a four year period (48 months).
- A student cannot be in more than one clique at any given time.
- Initially, all cliques must have at least one student.
- The coefficient variables of each rate of change represent the attractiveness of switching to a new group.
- The rates of change of the different groups sum to zero.
- The proportions of the population in each group sum to one.

A. Two Groups

We modeled two groups, $U$ and $C$, where $U$ is the “unaffiliated group” and $C$ is a clique. The rate of transfer between the two groups is denoted $\alpha$.

In the example below, we initially assumed the unaffiliated group contained 0.99 of the total population, while the clique contained the remaining 0.01. We made this choice because we assumed that the population was 100 and each clique must contain at least one member. We set $\alpha = 0.142$ to illustrate our model.

B. Three Groups

We adapted the two-group model to be a three-group model by introducing a second clique and expanding our differential equations to include the rates of transfer to and from the new clique. We modeled three groups, $U$, $C_1$, and $C_2$, where $U$ is the “unaffiliated group” and $C_1$ and $C_2$ are different cliques. We used six different variables to model the individual rates of transfer from one group to another, as displayed in the Markov Chain representation.

In the example below, we initialize $U = 0.98$, $C_1 = 0.01$, and $C_2 = 0.01$, for the same reasons as stated in the previous example. We set the coefficients to the values given below to illustrate our model.

### III. Comparison and Results

The three-group model shows great potential for fluctuation depending on the attractiveness of different groups. Groups can form rapidly and disperse just as quickly. In the three-group example, a clique began to form within four months. For that model, the value in each group should continue to fluctuate for a time period before one group has no remaining members and the model reduces to the two-group model. For the two-group model, the percentage of the population in each group will stabilize once one group obtains the entire population. We hypothesize this behavior will persist as we increase the number of groups.

Also, we hypothesize that with any number of groups, the Markov Chain representation will resemble the complete directed graph, with the number of vertices equal to the number of groups. This means the number of connections between the groups, and thus the number of coefficients, is equal to $n (n - 1)$, where $n$ is the number of groups and $n > 2$.

### IV. Reflection

This model does well in instances where the desired outcome is for one group to dominate. A major limitation of the model is the inability to adjust the attractiveness of a group over the given time period. For example, some groups lose their attractiveness as the size of the group increases, and our model cannot account for this. One resource we examined refers to this change as the utility of a group [2]. One of the disadvantages of this model was the large number of variables required to accurately model the situation. Every time another group was added the number of coefficients increased by $2 (n - 1)$ for all $n > 3$. Another disadvantage is that the number of groups is constant. In reality, large groups will break into smaller groups and smaller groups will be absorbed into other groups. Overall, our model suffers from a lack of flexibility and is unable to accurately model a dynamic real life situation.