Differential Equations and Recycling Efficiency

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The Problem:

When recycling began in the United States many places required that people separate the different kinds of materials they recycled, and the different materials were collected separately. The initial recycling rates were low, partly due to the inconvenience of having to separate and keep the materials apart. Since that time single stream recycling has become more common, and people can simply place all of their recyclable materials in a single bin. The resulting materials are then sorted at a recycling facility.
Recyclable materials generally go through a number of stages to separate the different materials. One stage is used for the materials that consist of either paper or cardboard materials. These are difficult materials to separate, and a good deal of this material is sorted by hand. The question to explore is whether or not a simple process can be developed that will help separate a large percentage of the materials, specifically paper and cardboard materials.
Potential Solution:

A simple device will be tested in which the materials will be dropped from a great height, and a fan will blow air across the stream of falling material. Determine the minimal height and wind speed that can be used to separate 30%-40% of the paper that is in the falling column of material. For our purposes you should assume that the distribution of the paper and cardboard items are relatively uniform but make sure your assumptions are explicitly stated. The goal is to establish the feasibility of the general idea before proceeding to a more complex situation.
\[ F_d = bv \]
\[ F_w = b(w - v) \]
\[ F_g = -ma \]
Position wrt y-axis

Due to the high drag characteristics of paper and cardboard we make the assumption that as the material enters the fan region it has reached terminal velocity.

Therefore:

\[ F_{net} = 0 \]

\[ ma = 0 \]

\[ my'' = 0 \]
\[
F_{net} = 0
\]
\[
ma = 0
\]
\[
my'' = 0
\]
Characteristic Equation:
\[
m\lambda^2 = 0
\]
\[
\lambda^2 = 0
\]
\[
\lambda = \pm 0
\]
Since this is a repeated root:
\[
y(t) = C_1 e^{0t} + C_2 t e^{0t}
\]
\[
y(t) = C_1 + C_2 t
\]
\[
y'(t) = C_2
\]
Position wrt x-axis

Recall Newton's 2nd Law: \( F = ma \)

Therefore:

\[
F = F_w \\
ma = b(w - v) \\
mx'' + bx' = bw
\]
\[ F = F_w \]

\[ ma = b(w - v) \]

\[ mx'' + bx' = bw \]

**Characteristic Equation:**

\[ m\lambda^2 + b\lambda = 0 \]

\[ \lambda(m\lambda + b) = 0 \]

\[ \lambda = 0, \lambda = -b/m \]

\[ x_c(t) = C_1 + C_2 e^{(-bt/m)} \]

Make the conjecture: \( x_p = wt \)

Therefore:

\[ x(t) = C_1 + C_2 e^{(-bt/m)} + wt \]

\[ x'(t) = \left( -\frac{bC_2}{m} \right) e^{(-bt/m)} + w \]
We make a couple of assumptions

\[ x(0) = 0 \quad \text{because the initial position on the x axis is 0} \]

\[ x'(0) = 0 \quad \text{because the initial velocity in the x direction is zero.} \]

\[ y(0) = h \quad \text{because the initial y position is the max height.} \]

\[ y'(0) = v_T \quad \text{because the initial velocity is assumed to be at terminal velocity} \]

\[ x(t) = \left( -\frac{wm}{b} \right) e^{-\frac{bt}{m}} + wt + \left( \frac{wm}{b} \right) \]

\[ y(t) = -v_T t + h \]
Citations:


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