Problem

In order to increase the efficiency of sorting recyclable materials, it has been suggested to drop paper and cardboard down a chute in the presence of a large fan. The aim of this model is to determine whether this is a feasible option, and if so, to minimize the height and wind speed required to separate 30-40% of the paper from the cardboard.

Development of the Model

Throughout the development of the model, certain assumptions were made. It was initially assumed that the air-stream of the fan acts on the object regardless of its vertical position. The fan was also assumed to have less effect on the falling object with a larger horizontal distance between the two. This relation was modeled with a linear equation. There were also various assumptions made about the properties of the cardboard and paper dropped. From research on the density and typical sizes of recycled paper and cardboard, a range of object sizes was approximated. It was assumed that the likelihood for all sizes in this range was equal. Consideration of the possible shapes of the material was also incorporated into these assumptions. For example, paper could be balled or flat, but cardboard is more likely to be recycled as a flat sheet. In the model, the random motion of a falling paper was considered small and therefore negligible. The air through which the object falls was assumed to be of a constant density and free of other significant air motion. The model was created under the consideration of two directions of motion with $x$ representing horizontal motion and $y$ representing vertical motion. The bins were assumed to be 1.9 meters wide, touching one another, and consistently emptied as to prevent a full bin. The tops of these bins was taken as the $y = 0$ position of motion. There were also practical limitations applied to the model. Due to data on the standard heights of American warehouses, a maximum height that gave sufficient room for the bins and other equipment was set at 7 meters. Similarly, fan research provided a typical speed range of 3 – 8 meters/second.

This model uses two forces acting on the falling objects: the force of gravity ($\vec{F}_g$) and the force due to air ($\vec{F}_a$). These values are given by

$$\vec{F}_g = -mg\hat{y},$$

$$\vec{F}_a = kv^2\hat{v},$$

where $g = 9.8$ meters/second$^2$ is the acceleration due to gravity, $k = \frac{1}{2}\rho C_D A$ is a constant that depends on the air density ($\rho$), drag coefficient ($C_D$), and cross-sectional area ($A$), and $v$ is the perceived velocity of the air within the frame of the falling object.

Separating these forces into their components and applying Newton’s Second Law in two dimensions,

$$\sum \vec{F} = m \left( \frac{d^2 x}{dt^2}\hat{x} + \frac{d^2 y}{dt^2}\hat{y} \right),$$

Figure 1: Free body diagram of falling object
the equations of motion are found to be

\[
\frac{d^2 x}{dt^2} = \frac{k}{m} \sqrt{\left( v_0 - cx - \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 \left( v_0 - cx - \frac{dx}{dt} \right)} + \frac{d^2 y}{dt^2} = -\frac{k}{m} \sqrt{\left( v_0 - cx - \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 \left( \frac{dy}{dt} \right)} - g,
\]

where \( v_0 - cx \) describes the air velocity due to the fan.

### Analysis and Results

Our model utilizes Mathematica to calculate the position of an object as it moved throughout the system. The \( x \) and \( y \) positions are calculated with respect to the previous values of position, velocity, and acceleration of the object. The model tracks the position of the objects through a small change in time, saving position values for each time step in the system, and then graphing these movements.

This model is set up for specific user inputs such as, initial fan velocity, the slope of the velocity with respect to distance from the fan, the height at which objects are dropped, as well as the amount of objects to be dropped in a specific test. The number of papers dropped vs the number of cardboard pieces dropped are all randomly selected when the model is initiated. Other values, such as the mass of the object and shape of the object are also randomly selected before the object is dropped in the system.

The analysis of this model came from another function that ran the model using a range of each of the initial conditions, and documenting the success of the model that was run. The model with the highest success rate shows the optimized height, fan velocity, and size of bins needed for a high sorting rate. To increase the reliability of the optimizations, we dropped more objects using ever tightening ranges of initial conditions, to determine the optimal value for fan velocity, height, and bin width. The optimum values were found to be 4.0 meters/second for fan speed and 0.9 meters for falling distance.

### Areas for Improvement

The relative simplicity of this model leaves much to be improved on and allows for many factors that go into development to be modified in a move towards a more complex system. A few of the areas this model can be improved upon are: a more elaborate assortment of shapes, sizes, densities, and configurations of materials; a larger quantity of data used in the analysis and optimization of the fan speed and height variables; a more in depth view of the natural conditions of the model, including a more variable air density and a random motion of air currents; a more complex system of dimensions to include a second horizontal movement for the objects; as well as consideration of more minute effects, such as fluid dynamics or Brownian motion.