One of the stereotypes about high school is the prevalence of cliques. Following this assumption, there is the question of how the social dynamics of these groups change over time. The goal is to be able to model this problem into an ODE, thus making it easy to predict these changes. There are a number of terms that are used to solve this that have to be defined. The first term is clique, which is defined as a group, either Clique x or Clique y, with a population of students. The second term is ”cool”, which is defined as the group that is growing at a positive rate. This is used to represent a clique that might be more attractive and gain new members. The third term is new member, which is defined as a student who transferred from one group to the other.

With these assumptions, there are other definitions that need to be laid out. The number of members in each clique is represented by values $x$ and $y$. The change in the number of members in a clique is represented by $dx/dt$ and $dy/dt$. Also there is the coolness factor, which is represented by the parameters $b$ and $c$ in the equations. Assumptions are made in determining the equations as well. For example, for the purposes of these equations the number of people in $x$ and $y$ is not changing, so $x + y = k$, where $k$ is a constant representing the total population of students. Based on research, another assumption is that people change groups based on traits, as opposed to people changing traits based on groups. Also, there are limits to how fast members can change groups. So for each $t + 1$, there can only be one member that changes groups. We started with a very simple model:

$$\frac{dx}{dt} = ax + by$$

$$\frac{dy}{dt} = cx + dy$$

In this scenario, $a$ and $d$ will only be positive if new students are entering the environment, or students are moving away, therefore in this case $a = 0 = d$. Based on which group is considered the cool group, the change in total clique members will be positive or negative as members leave to join the cool group. Consider the case when Clique x is the cool group. In this case, $b$ would be positive and $c$ would be negative as people would be leaving Clique y to join Clique x. The following equations show the solution to this equation.

$$x(t) = k_1 e^{\sqrt{|bc|}b} + k_2 e^{-\sqrt{|bc|}b}$$

$$y(t) = k_1 e^{\sqrt{|bc|}\sqrt{|bc|}} - k_2 e^{-\sqrt{|bc|}\sqrt{|bc|}}$$
By making use of the trace determinant plane, the determinant is a positive value and the trace is zero, so the resulting graph shows that \((0,0)\) is a center. As time goes on, the groups will return to initial conditions. However, in this case, a clique can never obtain a negative number of members, so the cool group will obtain the total amount of students, and the other group will contain zero members. Now if a third clique, Clique \(z\), were to enter the scene, the equations would become slightly more complex with numerous possibilities with the basic model of:

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\begin{align*}
\frac{dx}{dt} &= ay + bz \\
\frac{dy}{dt} &= cx + dz \\
\frac{dz}{dt} &= ex + fy
\end{align*}
\]

Constants \(a, b, c, d, e,\) and \(f\) will show which clique is considered the coolest of cool groups. If Clique \(x\) is considered cooler than Clique \(y\), but Clique \(y\) is cooler than Clique \(z\), then \(a, b,\) and \(d\) are all positive values and the remaining constants are negative. Another interesting situation to consider is the possibility that Clique \(y\) and Clique \(z\) are equally cool, but less cool than Clique \(x\). This results in positive values for \(a\) and \(b\), negative values for \(c\) and \(e\), and values of zero for \(d\) and \(f\). These provide more complex possibilities, but in the end, still result in all students landing in Clique \(x\). This model though a simple ordinary differential equation, gives the basic principles for the formation and deterioration of cool and uncool groups. However, the results are not all that accurate beyond the statement that everyone becomes acquaintances by the end.

This model does not fully consider all of the traits that group members may have more of a preference for over others, nor does it consider the complexities of drama that many middle schoolers and high schoolers face based on issues beyond that of personality traits. For example, by including a value \(xy\), the situation when one group member has all the say in which group is cool and may switch accordingly. Though this is a start of a possible model for social groups, no model will ever fully encapsulate the intricacies of personality, daily life drama, or personal interest change.