Problem A;  
Sorting Recyclables

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How do we separate paper and cardboard recyclables using a fan?

We will assemble a system of ODEs to model this scenario and find the necessary velocity and height of the fan.
Assumptions

- **No** air resistance
- Materials dropped from **the same height** and form a column with similar ratios and distribution
- \(D = 100\, \text{m}\)
- **Constant densities** for cardboard and paper
  - \(\rho_{\text{paper}} = 1.201\, \text{g/cm}^3\) (1)
  - \(\rho_{\text{cardboard}} = 0.698\, \text{g/cm}^3\) (2)
- **Constant areas** for cardboard and paper
  - \(A_{\text{paper}} = 20 \times 30\, \text{cm}\) (4)
  - \(A_{\text{cardboard}} = 25 \times 18\, \text{cm}\) (5)
- **Coordinate system** (next slide)
The height of the fan would need to be determined.

The strength of the fan would need to blow the paper horizontally 10 meters, without blowing the cardboard.

Paper and cardboard are dropped from same height, \( h_d \)

Paper is collected when it is blown horizontally 10 m.

Cardboard and residual paper are collected.

\[ h = ? \]
Components of the ODE

Our system breaks down the paper’s movement in two dimensions. The first describes horizontal motion and the second describes vertical motion.

\[ x'(t) + 10x(t) = 60v^2 \]

\[ x(t) = 6v^2 - 6e^{-10t}v^2 \]

\[ 9.8x''(t) + x'(t) = h \]

\[ -\frac{49}{5} e^{-\frac{5}{49}t} \left( \frac{177}{2} - h \right) + ht + \frac{8673}{10} - \frac{49}{5}h \]

In the first ODEs, it should be noted that the non-homogenous part comes from a wind velocity equation (3):

\[ F_w = \rho v^2 A \]
Additional Issue

Which aspect of your model results in the largest difference in sorting quality if that aspect undergoes a small change?

If the speed of the fan were changed even slightly, the balance of the sorting process could be overturned.

If the fan speed increased, it could move cardboard pieces out of the column with the paper.

If the fan speed decreased, it could fail to sort out the paper and all of the materials would fall straight down.
Conclusion

\[ x'(t) + 10x(t) = 60v^2 \]

\[ 9.8x''(t) + x'(t) = h \]

The 2 ODE's above describe the **horizontal and vertical trajectory** of the paper.

The **velocity** of the fan needs to be 1.6 m/s and that the **height** of the fan needs to be 550 m.

A change in **fan speed** would result in the most significant difference in sorting quality.
References

(1) Paper Density, Aqua-Calc


(2) Cardboard Density, Aqua-Calc


(3) Wind Velocity Equation, Engineering Toolbox

https://www.engineeringtoolbox.com/wind-load-d_1775.html

(4) Paper Size, Design Resources


(5) Cardboard Size, Start Solid

https://www.startsolid.com/pages/a-catalog-of-amazon-box-sizes