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& OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

AUTHOR COMMENTS ON SCUDEM II 2018 PROBLEMS

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1 Overview

For the SCUDEM II 2018 Student Competition Using Differential Equations Modeling student teams were asked to choose one question from a set of three, and they were expected to develop a mathematical model that can be used to provide insights into the question they picked. The first question required students to determine whether or not a method to separate paper using a fan is feasible. The second question required students to determine how an animal can use the perceived size and rate of change of the perceived size of another animal to determine if the other animal represents a threat. The third question was to determine how the composition of social groups or cliques evolve in time.

This year a larger number of teams, took part in the event. Forty teams chose to examine Problem A, nineteen chose to examine Problem B, and twenty-two examined Problem C. Compared to the previous year, teams this year provided more modest models, which is a good thing. We do not expect complicated models for a first attempt at a new model, but rather we hope to see teams demonstrate good modeling practice. The practice of developing mathematical models requires a small amount of discipline and a large dose of flexibility and humility. In particular, it requires teams to create a model using a good dose of creativity and good practice, take a critical look at a current version of the model, use discipline in analyzing the model, practice humility in accepting the parts that should be changed, and be persistent in repeating the whole process.

Each student team submitted two documents. The first document consists of the slides from a team's Presentation. The second document is a brief Executive Summary. We offer a response for each of the documents. First some notes and observations are provided for the Presentations, and then some notes and observations are provided for the Executive Summaries.

2 Presentations

The first observations are for the presentations. The slides from the presentations do not provide a complete story, but they do convey a sense of a team's efforts. Some notes are provided about the slides. After the observations about the notes are given some general comments are given about the presentation of the analysis of a mathematical model.

2.1 Observations

Presenting mathematics is a difficult task. A team cannot provide all information associated with their efforts, and the team members must decide which parts on which to focus. Here I will note a few aspects that stood out, starting with some general trends that could be improved. Next I will discuss some of the more positive aspects.

The first thing that stood out was that many teams provided a set of slides without an overview and without much space dedicated to discussing the context. Preparing your audience and providing the background is an important part of a presentation. It should be assumed that most of your audience is not familiar with the situation and do not know the original question.

Another aspect to note is that many slides had a great deal of verbiage and were relatively busy. A slide in a presentation should not be read directly but provide an outline for the speaker. It is there to support the discussion. You want your audience to listen to what is being said rather than reading the slides. Each slide should only have a few points with direct, simple language.

The last thing to note on the downside is that a number of presentations had slides that simply had long equations without much else on them. This is going a bit too far in the minimalist direction. A better balance is necessary to help the audience understand the motivation for why a team chose specific terms in their systems.

On the plus side, a number of teams were careful to present carefully laid out conclusions with justifications. This is a difficult task that challenges experienced professionals. Results can be given in a variety of forms, and some teams used combinations of plots, tables, and written descriptions in creative ways.

As noted above some teams provided systems of equations with little background or motivations, but a number of other teams provided introductory slides that did examine the basic ideas behind their work. In particular some teams carefully described the fundamental principles that were used to develop models from first principles. This was expected for the first problem, where falling paper could be described by first applying Newton's Second Law. To my surprise, though, a number of teams examining the third question, formation of small social groups, did a terrific job of discussing their motivation and discussing the various terms used in their model. I expected to see approaches based on infectious disease type models. Many teams took this approach, and a good number were careful to lay out the motivation for their choices.

2.2 Analysis of a Model

One aspect of the presentations that had a wide variation was in the analysis of the models. Given the time constraints and the students' level of mastery we expect relatively simple models. The first step in the modeling process is to try to start as simple as possible and try to capture the basic nature of the phenomena of interest as a way to see which parts of the system have the biggest impact. The model should then be evaluated and critiqued and modified as deemed necessary.

The contest is not so much about the model but is about the modeling process. We want to see if teams can *model* how to do good modeling. A crucial aspect of modeling is tearing apart the mathematical approximation and determining how it can be improved.

As an example, a number of teams provided approximations to their models under different assumptions or different values of the parameters. A number of teams that examined the second problem, the flight response in prey animals, provided plots of solutions to their system. Most notably, they compared their solutions with some of the plots given in the original paper. Some of these plots demonstrated the same general trends as the plots in the paper, but many had different features. Whether or not they match the original paper, though, is not as important as recognizing that the comparison should be made, and it is impressive to see such comparisons.

Another example comes from the first problem, falling paper products. One of the things that surprised me was that few teams focused on the idea of how the papers would be distributed as they hit the ground. A small number of teams did address this critical part of the problem. Teams that provided histograms and a careful description of how they generated data provided remarkable insight into the problem and also managed to present their data in an insightful and direct manner.

As noted above, I was surprised to see the kinds of analysis that were provided for the third question about social groups. My expectations were to see teams borrow heavily from the infectious disease literature and look at numerical results. A number of teams, though, went further and discussed the stability of the solutions including determining when the solutions might result in oscillations, growth, or decay.

More importantly, some teams took part in an explicit sensitivity analysis of their model. This is another crucial aspect of the modeling process. To examine what happens to the nature of the model under small changes of a parameter or varying an assumption gives important insights into which parts of the model can yield different kinds of behaviors. It also helps the modeler understand which terms in the model have the biggest impact under different situations.

3 Executive Summaries

The other requirement for the teams was to submit an Executive Summary rather than a full report. To be able to write an executive summary is an important task, and many undergraduate students have few opportunities to practice this craft. In this section I will provide a few observations about the executive summaries. I will also provide a broad overview of what we generally expect to see in an executive summary.

3.1 Observations

One common feature of many of the executive summaries was the inclusion of the differential equations that a team developed to approximate the given context. In some cases the teams defined the variables, but in other cases it was not clear what quantities the variables were used to track in their model. Additionally, in some summaries it was not clear if the variables were scalar or vector quantities, while in other papers the teams were careful to denote the nature of the quantities and were careful in using consistent notation.

Another source of variation was in how the teams discussed the motivation for their model. For example, some teams stated the equations without describing the meaning of the terms in the equations. Other teams carefully stated what rates their model was tracking and how the individual terms mimicked different behaviors relevant to the situation. Teams that described the model and provided some context for the model made a more favorable impression and helped the reader understand some of the choices the team made.

Finally, another source of variation was the level of detail provided about a team's results. Some teams did not discuss their results nor provided details into the general behavior and overall trends that the solutions to their model might exhibit. Other teams did a good job of discussing the solutions and general trends. In particular, some teams provided graphs that gave a feel for how the solutions changed for different values of a parameter. Most notably, several teams discussed the overall stability of their model with respect to a couple parameters. For example, one team provided insights into the general nature of the solutions of their model based on the trace-determinant plane.

3.2 Writing an Executive Summary

As noted above writing an executive summary is a non-trivial task. Unfortunately, it is a task that many students (at any level) are given little to no opportunity to practice. In this section I provide some insights into what I personally expect to see when reading an executive summary. The important thing to keep in mind is that the reader may only have a passing familiarity with the topic, and you have to be careful to provide a general context for the reader.

The first thing that is expected is that the basic problem and the context for the problem should be stated. The reader may not be familiar with the original question. The first one or two paragraphs should make the context clear, and it should not be assumed that the reader was fully aware of the situation prior to picking up the executive summary.

The second thing that is expected is that the reader will not be able to read a set of equations and understand what they mean or why they are used without some guidance. The variables should be briefly defined, and proper notation should be used. Some motivation should also be given before presenting the equations. For example, in an equation describing how the composition of social groups change, a team may decide to multiply two terms together as a way to approximate the rate that members from two different groups may intermingle and hence switch groups. The phenomena of mixing should be briefly discussed and the terms referring to that mixing can be explicitly stated.

Because of the strict page limit, though, it is not expected that there is much detail beyond simply stating the phenomena and associated terms.