Differential Equations
SIMIODE Learning Environment  Index

Index of Modeling Scenarios and Text Narratives
Numbered materials have been refereed, revised, and posted.

This Index is meant to simulate the topics found in a traditional differential equations course.
The numbering system reflects chapter sequencing.

These materials are hyper-linked to Student Versions under Resources at https://www.simiode.org/
and they are available to the public with no sign in or membership.

User must join and be registered and logged in to Teachers Group
to see Extended Teacher Versions.

Those materials not hyper-linked are under development.

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and on Modeling Section heading to return to Table of Contents.
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APPENDIX
Strategies for Analytic Solutions and Qualitative Analysis

First Order Differential Equations
Second Order Differential Equations
Linear Systems of Differential Equations
NonLinear Systems of Differential Equations
Fourier Series Development
Partial Differential Equation Solution Strategies
Modeling One - First Order Differential Equations

Narrative and Development
Modeling Death with M&M 's and Simulation
Immigration Model with M&M's
Modeling Change Discretely
Moving to Differential Equation Model
Introducing CAS solving with modest analytic solutions in appendices as needed

1-001-Text-SepVariables
   Technique narrative on solution method of separation of variables

1-002-Text-Integrating Factor
   Technique narrative on solution method of integrating factors

1-009-Text-Bifurcation
   Early introduction to bifurcation with experimentation.

Activities

1-001-M&MDeathImmigration
   Modeling Death and Immigration with M&M 's and Simulation

1-001A-M&MDeathImmigration-Variation
   Variation on modeling death and immigration with M&M’s

1-001B-M&MDeathImmigrationMystery
   Each student sets own immigration rate and others need to solve this mystery

1-001pgf-BirthDeathImmigration
   Probability generating function approach to simulation.

1-002-Tossing
   Modeling a simulation of a large number of dice tossings

1-003-CollegeSavings
   Saving for Child's College Education

1-004-MicroorganismImmigration
   Modeling Immigration in a Petri Dish

Evaporation and Radioactive Decay Modeling
Modeling Falling Stack of Coffee Filters
Modeling Sphere of Salt in Solution

1-005-OilSlick
   Modeling the Spread of Oil Slick with Incomplete Data

1-005A-OilSlick
   Same as 1-5-OilSlick but with client setting.

1-005a-ChemDataCollection
   Analysis of incomplete reaction data
1-005b-ChemDataCollection
Analysis of incomplete reaction data

1-006-FinancingSavingsAndLoan
Bank Investment Analysis and Bank Loan Analysis

1-007-AntTunnelBuilding
How long does it take an ant to build a tunnel of length x in soil?

1-009-ICUSpread
Modeling the spread of ICU’s in US Hospitals from 1958-1974

1-010-LSDAndProblemSolving
Modeling LSD in the body and correlating amounts with test performance.

1-011-Kinetics
Chemical Kinetics Models - Zeroth, First, and Second Order Reactions

1-011A-Kinetics
Same as 1-11-Kinetics but with more guidance and less narrative.

1-012-SublimationCarbonDioxide
Sublimation of Carbon Dioxide

1-013-Sleuthing
Situations associated with stopping cars, projectile steel ball, and time of death

1-014-DrainingContainers
For fixed volume column which radius of cylinder of water drains fastest

1-015-Torricelli
Modeling the Emptying of a Column of Water

1-016-DogDrugs
Modeling drugs for anesthesiology

1-017-DiseaseSpread
Modeling spread of disease using logistic equation

1-018-LogisticPopModeling
Limited Growth Population Modeling

1-019-RocksInTheirHeads
Sensitivity Analysis - Data Collection Experiment Comparing of Rock Masses

1-020-IceMelt
Analyzing models on which melts first a sphere or cube of ice of the same volume

1-021-FeralCatControl
Model for three feral cat control policies

1-022-SpreadOfTechnology
Model the spread of a number of technological advances

1-023-RumorSpread
Rate of Spread of False and True articles on the Internet

1-024-MalariaControl
Modeling and numerical methods for first order malaria growth
1-025-MixingItUp  
Modeling more and more complex salt mixing situations.

1-026-Evaporation  
Modeling the evaporation of an alcohol and water mixture in various containers.

1-027-StochasticProcesses  
Modeling randomness with stochastic processes.

1-028-SouthernSweetIcedTea  
Data is offered to model making sweet iced team using luminescence

1-029-ConeToCubeFlow  
Modeling water flowing from cone to cube and out

1-030-IntraocularGasBubbles  
Modeling dissipation of intraocular gas bubbles used in eye surgery.

1-031-CoolIt  
Changing temperature of container of water in a changing environment.

1-032-WordPropagation  
Modeling the rate at which words propagate through English language text

1-033-SouthernBarbeque  
Phases of barbequing brisket are modeled using real data.

1-034-FishMixing  
Student designed fishing strategies for mix of fish in lake

1-036-NeutralBuoyancy  
Finding depth in water at which an object settles to neutral buoyancy

1-037-CommonColdSpread  
Students conduct simulation of spread of common cold and model.

1-038-Ebola  
Modeling Ebola epidemic with first order differential equation models

1-040-OutcomeSavings  
Determining monthly deposit rate for long term savings goal

1-041-AirToTop  
Variable ascent rate and air management in SCUBA diving

1-042-Kool-Aid  
Modeling the amount of drink powder in a second tank of flow system.

1-043-CoolingUpAndDown  
Air conditioning cooling modeling.

1-044-CollegeBound  
Planning for full college education costs for the daughter of a friend.

1-045-TimeOfDeath  
Determining time of death given observations and environmental conditions.

1-050-BargeAhead  
Optimizing a barge trip upriver
**1-051-OneTankSaltModel**
Build one compartment salt mixing model

**1-052-SaltWaterTanks**
Studying tank in which water inflow containing salt increases.

**1-055-WaterFallingInCone**
Modeling the falling of water in a right circular cone

**1-057-FiguringFluidFlow**
Evaluating three models of fluid flow from a tank using data

**1-058-WaterClocks**
A container is designed so water will fall out at constant rate of change in height.

**1-059-ContainerShapeFallingWater**
Modeling column of falling water in different shaped containers.

**1-060-SalesMarketing**
Building a model of sales of consumer products from a classic marketing study.

**1-064-TorricelliBox**
Modeling falling column of water with a box at the base of the column

**1-061-PotatoCooling**
We model cooling of a baked potato and compare it to student-collected data.

**1-066-USCensusModeling**
Modeling the US Census data with several different models.

**1-070-FisheryHarvest**
Modeling harvesting of Atlantic cod fishery

**1-071-S-NewtonWatsonTimeOfDeath**
Sherlock Holmes determines time of death

**1-073-WaterExitBottle**
Estimating a parameter in Torricelli’s model of water exiting a container

**1-074-BottleWaterFlow**
Comparing two models of water flowing out of a container through exit hole

**1-079-HomeHeating**
Modeling how to heat your home while you are away

**1-080-DrugAdministration**
Building a simple model for drug administration

**1-081-TumorGrowth**
Two different models for growth of cancer tumor.

**1-085-DrugBolus**
Modeling intravenous bolus of drug in the body

**1-086-MedicinalPill**
Modeling administration of medicinal pills.

**1-090-EmptySphericalTank**
Comparing two ways to empty spherical tank of water
1-091-Slopefields
   Building population models for various situations and using slope fields
1-093-SucroseReaction
   Determining model for sucrose hydrolysis using lab data
1-094-SteepingTea
   Modeling temperature change and dissolution of sugar in brewing fruit tea
1-095-RatingChessPlayers
   Using Elo’s Method for rating chess players and difference equations
1-101-ClassM&MDeathImmigration
   Generating data and using individual model to estimate parameters
1-105-AnimalFalling
   Comparing terminal velocity for variety of animal’s falling
1-115-ModelingWithFirstOrderODEs
   Several short illustrations and two exercises for modeling
1-120-CircularRollerCoaster
   Modeling a circular roller coaster to determine velocity to stay on track
1-122-SpreadPEV
   Using recent sales data model the spread of plug-in electric vehicles.
1-140-LeakyBucket
   Modeling the height of water in a tank with a leak and water pouring in
Narrative and Development
Differential Equations Which Are Just Impossible!
Simple Step Iterating Method
Euler's Method
Numerical Experiments
Improving on Euler's Method
Comparison of Methods with CAS DE solver approach

Activities
These are embedded in the development of the numerical methods.
**Modeling Two - Limited Growth Models**

**Narrative and Development**
Exponential Growth Population Model  
Limited Growth Population Modeling  
Logistic Model Development  
Estimating Parameter Strategies for Logistic Growth Model

**Activities**
Comparison with Transformed Data vs. Direct Optimization of Sum of Square Error Approaches for Logistic Growth in Parameter Estimation  
Simulation with M&M's of Logistic Growth Model for Spread of Disease  
Analysis of "Struggle for Existence" Data from G. F. Gause, Soviet Ecologist, From 1930's  
Modeling the Spread of Technological Innovation in the United States with Real Data  
Maximum Sustainable Yield In Harvesting Models  
Harvesting a Renewable Resources - Some Analysis  
Running a Catfish Farm
Modeling Three - Second Order Homogeneous Differential Equation Models

Narrative and Development
Spring Mass Dashpot Modeling
Modeling with Newton's Second Law of Motion
Free Body Diagram Modeling Tool
Using Initial Condition Information for Completion of Solution
General Forms of Solutions
Characteristic Equation and Roots and Their Significance
Complex Roots to Characteristic Equation and Their Meaning
Various Types of Damping - Under, Critical, and Over
Repeated Roots for Characteristic Equation - Self-Discovery
Bookkeeping and Rearrangements - Phase Angle Issues

Activities
3-001-SpringMassDataAnalysis
  Data on a spring mass system with resistance is given for modeling for analysis
3-002-ModelsMotivatingSecondOrder
  From real data several ways to model spring mass system emerge.
Estimating Resistance Parameter in Spring Mass Dashpot Modeling
  Design
Modeling Parachuting
Modeling Frequency of Spring Mass Dashpot Motion
3-004-VanderPol
  Study of van der Pol’s equation with applications and spreadsheet simulation
3-006-Buoyancy
  Data on a bobbing container motivates model and parameter estimation
3-008-HangTime
  Hang Time Modeling
3-010-EnergyInSpringSystems
  Exploring damping and forcing terms to discover energy in system
Logarithmic Decrement
Bad News - Positive Real Part of Characteristic Root
3-013-WhiffleBallFall
  Using data on whiffle ball fall model resistance and predict the fall position.
Rocket Thrust Modeling
Inverse Problems - Ascertaining Parameters Given Observational Data
3-016-FallingCoffeeFilters
  Using data on stack of coffee filters to build model and estimate parameters.
3-017-StackedCoffeeFilters
  Using data on stacked coffee filter falling from the literature build models.
3-019-ShuttleCock Fall
Modeling a falling shuttlecock

Keeping Costs Down - Manufacturing Cheaper Springs with High Performance

RLC Circuits - Basic Circuit Notions and Second Order Differential Equation Models

Swinging Along - Modeling, Massless and Physical Pendula

Pendulum Motion and Logarithmic Decrement

3-029-FerrisWheelCatch
We model the throw of an object to a person on a moving Ferris wheel.

3-030-SecondOrderIntro
Broad introduction to second order homogeneous and nonhomogeneous, linear, constant coefficient, differential equations with many applications

3-035-StadiumDesign
Design a stadium which is fair to home run hitters in all directions.

3-040-FirstPassageTime
We model and determine the first passage time for underdamped oscillator

3-041-UpDown
Relate times when projectile passes the same point – up and then down

3-044-DeepWell
Given total time of pebble fall to sound coming back tell how deep a well

3-045-RampBounce
Bounce a ball on a tilted ramp for optimal horizontal distance traversed

3-054-Relay
Place infielder in optimal position for minimum time relay through form outfield

3-065-UpDown
Modeling vertical projectile motion with resistance to address some issues.

3-070-FallingInWater
Drop a canister in column of water, collect data from video, model motion

3-075-RLCCircuits
An introduction to RLC circuits is offered including definitions and modeling.

3-080-PendulumModeling
Several different pendulum configurations are modeled and compared.

3-085-SimplePendulum
Modeling pendulum motion and verification of period with data

3-090-OneMassSpring
Data on a single mass spring system permits modeling of oscillator

3-090-Text-ChebyshevPolynomialSolution
Small study of Chebyshev Equation for which there are polynomial solutions

3-101-SpringMassFirstTry
Modeling a simple spring mass with no damping conjecturing solutions
3-102-SpringMassDamped
   Modeling a simple spring mass with damping conjecturing solutions
3-110-MilitarySpringMassApplication
   Modeling the shock absorber system for an Army vehicle.
3-140-TwoSpringOneMassFixedEnds
   Modeling two spring, single mass with fixed ends
Modeling Four - Second Order NonHomogeneous Differential Equation Models

Narrative and Development
Driver for Spring Mass Dashpot Modeling
Intelligent Conjecturing for NonHomogeneous Solution
Building General Solutions with Homogeneous and NonHomogeneous Parts
Transient and Steady State Portions of Solutions
Phase Angle for Solutions
Using Initial Condition Information for Completion of Solution
General Forms of Solutions

Activities
RLC Tuner Circuit to Build a Radio
Parameter Estimation Through Steady State Data
Forced Vibration and No Damping
Frequency Response - Maximum Steady State Amplitude

4-020-AnIEDBlast
  Modeling the effects of an Improvised Explosive Device

4-023-MysteryCircuit
  Students assigned various input voltages to a circuit to see what the circuit is.

Building Swaying
Killer Speed Bumps

4-035-ParEstSteadyState
  Input Output Analysis analyzing steady state to estimate parameters.

4-050-ResonanceBeats
  We study the notions of resonance and beats for undamped system

4-060-CircuitTuner
  Building the differential equation for a radio tuner.
Modeling Five - Linear Systems of Differential Equation Models

**Narrative and Development**
Intelligent Conjecturing for Homogeneous Solution
Eigenvalues - Voodoo Dispelled
Converting Second Order to System and Meaning of Eigenvalues and Eigenvectors.
Building General Solutions with Homogeneous and NonHomogeneous Parts
Time Out for Small Motor Skills and Eye Hand Coordination Practice
Trial Run at Two Solution Strategies
Two Compartment Model Analyses

**Activities**

*5-005-Dialysis*
Modeling Dialysis Machine

*5-007-ChemOpt*
Optimization for a Chemical Reaction

*5-0012-LipoproteinMetabolism*
Medical study data to build and affirm build a model for low-density-lipoprotein.

**Tuned Mass Dampers**

*5-014-TwoSpringMass*
Build Free Body Diagram and model for two spring configuration.

*5-022- ColdPill*
Modeling flow of drug from gastrointestinal tract to bloodstream for peak

**Optimization for a Chemical Reaction**

*5-028-SaltCompartments*
Amount of salt in two water tanks is modeled in several ways

*5-030-AirshedSulphur*
Analyzing a model of the production of sulphur compounds in a Montana airshed.

*5-040-TunedMassDampers-Part I*
Applying a second mass to keep structure from experiencing resonance

*5-040-TunedMassDampers-Part II*
Applying second mass to keep structure from large displacement

**Inverse Problems - Ascertaining Parameters Given Observational Data**

*5-080-SpaceFlightRecolonize*
Modeling the recolonization of the human race on a distant planet.
Narrative and Development
Numerical Solution Strategy
Equilibria and Stability Analysis
Linearization and Support from Homogeneous System Analysis
Nonlinear Ecological Modeling
Predator-Prey Modeling
Mimicry in Nature
Competition Modeling
Higher Trophic Level Models
West Point Acorn, Rodent, Rattlesnake Populations
Boys Dormitory Epidemic
1914 Influenza Epidemic
Epidemic Models
Flour Beetles Predation with Hiding

Activities

6-001-Epidemic
   English Boarding School NonLethal Influenza
6-002-EulerCromerPendulum
   Using the study of nonlinear pendulum to implement numerical methods
6-004-VillageEpidemic
   Build a model of a mid seventeenth century English village epidemic

Competition in Ecology

6-005-InsectColonyOpt
   Insect Colony Optimal Control
Predator Prey Modeling with Satiation and Limited Growth
Predator Prey Modeling with Hiding
Predator Prey Modeling and Optimal Control

6-008-PursuitModels
   Linearization and Support from Homogeneous System Analysis

6-012-RiverCrossing
   Building a model to help cross a river with current to land at specific spot
6-015-CombatingEbolaEpidemic
   Making policy recommendations from models of spread of Ebola

6-018-ExploringSIRModel
   Modeling rumor and disease spread.
6-020-AlgaePopulationSelf-Replenishment
   Investigate the massive algal blooms that struck Lake Chapala, Mexico
6-021-AcornsRodentsSnakes
Building a three trophic level model of acorns, rodents, and snakes

6-023-DroneHeadingHome
Moving against a headwind create model of drone flight to fixed delivery point

6-025-WhalesAndKrill
Use Excel to observe qualitative behavior a predator-prey model

6-028-SaltCompartments
Amount of salt in two water tanks is modeled when tank volumes are changing

6-030-SaltTorricelli
Modeling complex salt levels in a falling column of water.

Nonlinear Ecological Modeling

6-035-Shampoo
Modeling the amount of shampoo in a bottle during a shower

Predator-Prey Modeling

Competition Modeling

6-040-StruggleForExistence
Using historical data to model multiple species growth

6-070-BeerBubbles
Modeling the rise and size of beer bubbles in a sitting glass of beer
Narrative and Development
Modeling with Spikes and Jumps
Transformations in General
General Laplace Transforms - Lots of Functions
Revisit Second Order Differential Equations and Linear Systems with Laplace Transform
Transfer Function Thinking
Solution Strategies with Laplace Transforms
Living in the Frequency Domain
Convolution Applications

Activities
7-005-LaplaceTransformOverview
Introduction and application of Laplace Transforms

7-008-MachineReplacement
Laplace Transforms - Convolution Applications - Replacement Theory
RLC Filter Circuit and Laplace Transform View Point

7-010-MultipleDoses
Modeling several multiple dose approaches for drugs is considered.
Narrative and Development

Representing functions as sums of trigonometric functions.

Discovery and Least Squares Criteria
Approximating Functions with sums of Sine Functions
General Fourier Series Modeling

Activities
Creating Complex Sounds from Simple Sounds
Analyzing Signals with Simple Functions
Orthonormal Families and Their Good Times
Modeling Nine - Modeling with Differential Equations in Higher Dimensions

Narrative and Development
Partial Differential Equations
Numerical Solutions
Modeling Spread of Heat in Limited Environment - Heat Equation
Applications of the Heat Equation - Formulating Conditions
Extensions of the Heat Equations
Making Sound with Waves - Wave Equation
Modes
Analytic Solutions - Building for Success
Boundary Value Problems and Fourier Analysis - Bringing it All Together

Activities
Root Cellar Modeling with Heat Equation
Noninvasive Analysis with the Heat Equation
9-012-PDEGuitarTuning
   Tuning a Stringed Instrument with the Wave Equation
Orthonormal Families and Their Good Times
9-020-HorizontalBeam
   Modeling a suspended beam and collecting data to justify the model
Modeling Ten – General Modeling

APPENDIX

Strategies for Analytic Solutions and Qualitative Analysis

**First Order Differential Equations**

1-001-Text-SepVariables
1-002-Text-IntegratingFactor
Conjecturing
1-009-Text-Bifurcation

**Second Order Differential Equations**

Conjecturing and Eigenvalues
The Cases for Eigenvalues
Homogeneous and NonHomogeneous Solutions and Building Final Solutions
3-090-Text-ChebyshevPolynomialSolution

**Linear Systems of Differential Equations**

Equilibrium and Stability Issues
Conjecturing, Eigenvectors, and Eigenvalues
Construction of Solutions - Homogenous and NonHomogeneous Equations
5-030-Text-LinNonHomoSystemSol

Variation of Parameter Method for linear system of nonhomogeneous equations

**NonLinear Systems of Differential Equations**

Equilibrium and Stability Issues
Linearization and Translation
Some Special Situations - Orbits in Predator Prey Models

**Fourier Series Development**

8-002-Text-TrigSumRepresentation
General Formulae
Extensions - Odd and Even
Orthonormal Family Theory

**Partial Differential Equation Solution Strategies**

Separation of Variables
Boundary Value Problems
Appearance of Fourier Series