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STUDENT VERSION
Growth of Farmland

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STATEMENT

One area of early civilization is in ancient Mesopotamia (3500 B.C. and later), which is in the now Iraqi region of the Tigris and Euphrates rivers in the Middle East. These civilizations arose and expanded through the invention of agricultural practices. Archeologists would like to have an accurate description of the amount of land farmed over time to see if it might correlate with climate patterns, wars, or other factors. They have acquired much of our knowledge about these civilizations by excavating ancient ruins, analyzing writings in various forms, and examining relics that still survive. From some of the writings they have found, they have gathered data on the amount of area farmed 10 years apart from each other. But because of disturbances of the sites over time, they cannot tell how far apart these pairs of data are from each other. Table 1 shows the data we have, from earliest to latest:

In this project, you will have two submittals:

Draft Model Report: After you collaboratively work on developing your model(s) of this scenario in parts of two class sessions, you will submit your introduction to the problem, as well as your draft differential equation model(s) and a full description of how you arrived at the model. The purpose of this submittal is to ensure you have an accurate model before moving forward and also to receive feedback on that model. See the rubric regarding the introduction and the model description.

Final Report: After receiving feedback on your model(s) from the instructor, you will submit a final report where you mathematically analyze the model (qualitatively, analytically, and numerically), make predictions, interpret your predictions and conclusions, evaluate those

A(t)	A(t+10)
10.0	10.3
20.5	21.6
25.8	27.6
44.4	50.0
66.7	80.0
100.0	133.3
160.0	266.7
200.0	400.0

Table 1. Amount of Farmland in Unknown Year & Amount of Farmland 10 Years Later

conclusions, and reflect on the project experience. See the rubric for the detailed requirements. Here are some pertinent questions that provide specific ideas for prediction that you should address and you may have more of your own to add:

1. Plot your solution of your model of the farmland area as a function of time.
2. Predict the future size of the farmland, say at t = 50 years, t = 100 years, t = 150 years, and t = 200 years using your analytical, graphical, and numerical methods.
3. Find the time at which the farmland area is 150 hectares using your analytical, graphical, and numerical methods.
4. Determine the time of the first, fifth, and last observations above.
5. Create your own question that is interesting and that you can answer from your model, and, of course, answer it.

Be sure to utilize the rubric as your guide in writing your final report.

The major elements of the rubric are:

Model development: Improve or correct the draft submittal, if applicable, with explanations of the model development.

Mathematical analysis: Solve the model qualitatively, analytically, and numerically. This area of the rubric also entails comparing and contrasting the different solution techniques.

Predictions: Make predictions from the solutions, including specific predictions requested in the project directions.

Interpretations: Interpret the predictions and conclusions, which means to discuss those predictions in terms of the applied problem, not simply providing the mathematics.

Evaluation: Evaluate the accuracy and value of those predictions and conclusions by critiquing the interpretations and predictions. Also discuss the big picture impact of the results.

Reflection: Reflect on the project experience. This element requires looking back on the process to ponder what was learned, mathematically and non-mathematically (such as work habits or attitudes), and what connections were made by doing the project. It also entails looking forward toward other possible investigations that could arise from this project.