M&M Game Revisited

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STATEMENT

It is universally assumed that the probability of an M&M chocolate, when tossed, falling with the M side up is $\frac{1}{2}$. For this reason the M&M game [1] results in the stable population of $2Q$, where $Q$ is the number of M&M’s added at each time step.

As a reminder of the game in [1] we quote the rules:

Gently shake the M&Ms out onto the desk (you might want to use a paper plate to catch the M&Ms and keep them clean as well). We determine for each M&M if it lives or dies. If the M shows on top or up this M&M dies, otherwise there is life for this M&M. Upon death you should remove the M&M from the population (set these aside as we will need them for another experiment), count and note down the number of M&Ms who survive, and thus put fewer M&Ms back into your container for the next iteration. Do this over and over and record your results.[1]

The goal here is to find the probability distribution function (pdf) for the probability, $q$, that

$$Pr(\text{one randomly chosen M&M falls with M up when tossed}).$$

How to play and collect data

Using a pack of M&M’s and play the M&M game [1] many times, say 30 times and report what you believe is a stable value from these outcomes. The difference formula for this activity is given in

$$P_{n+1} = qP_n + Q \quad \text{where} \quad \begin{cases} P_n & \text{number of M&M’s at time step } n \\ q & Pr(\text{one randomly chosen M&M falling M up}) \\ Q & \text{number of M&M’s added at each time step} \end{cases} \quad (1)$$
The stable solution is when \( P_{n+1} = P_n = P \) for large \( n \). This results from (1) and analysis show:

\[
P = qP + Q \implies P(1-q) = Q \implies P = \frac{Q}{1-q}.
\]

**Example**

Suppose there are 15 students in the class and each one reports the stable number after 30 tries. With \( Q = 10 \) the results are given below for students \( S_i, i = 1, 2, \ldots, 15 \).

<table>
<thead>
<tr>
<th>Student</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
<th>S9</th>
<th>S10</th>
<th>S11</th>
<th>S12</th>
<th>S13</th>
<th>S14</th>
<th>S15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>22</td>
<td>22</td>
<td>23</td>
<td>19</td>
<td>19</td>
<td>19</td>
<td>24</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>21</td>
<td>22</td>
<td>24</td>
<td>18</td>
</tr>
</tbody>
</table>

We have

\[
P_{\text{stable}} = \frac{Q}{1-q} \implies q = 1 - \frac{Q}{P_{\text{stable}}}.
\]

Using frequencies and the above formula we end up with the probability mass function (pmf) or probability density function (pdf) for \( q \):

\[
f(q) = \begin{cases} 
2/15 & q = 8/18 \approx 0.4444 \\
4/15 & q = 9/19 \approx 0.4737 \\
1/15 & q = 10/20 = 0.5 \\
2/15 & q = 11/21 \approx 0.5238 \\
3/15 & q = 12/22 \approx 0.5455 \\
1/15 & q = 13/23 \approx 0.5652 \\
2/15 & q = 14/24 \approx 0.5833
\end{cases}
\]

Using this we can find the probability of a random M&M falling with M up in the following way:

\[
q = \Pr(\text{one randomly chosen M&M falling M up}) = \sum_{k=18}^{24} \Pr(\text{one randomly chosen M&M falling M up} \mid q = 1 - 10/k) \cdot \Pr(q = 1 - 10/k) \\
= \left( \frac{8}{18} \right) \left( \frac{2}{15} \right) + \left( \frac{9}{19} \right) \left( \frac{4}{15} \right) + \left( \frac{10}{20} \right) \left( \frac{1}{15} \right) + \left( \frac{11}{21} \right) \left( \frac{2}{15} \right) + \left( \frac{12}{22} \right) \left( \frac{3}{15} \right) \\
+ \left( \frac{13}{23} \right) \left( \frac{1}{15} \right) + \left( \frac{14}{24} \right) \left( \frac{2}{15} \right) = 0.5133.
\]

**ACTIVITIES**

**Activity 1**

Derive (1) with explanation.
Activity 2

Conduct your own experiment with classmates leading to your estimate of $q$, the probability of a random M&M falling with M up. From your results do you thin it is a reasonable assumption that the probability of a random M&M falls with M up is $\frac{1}{2}$?

REFERENCES