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SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

STUDENT VERSION
ANT TUNNEL BUILDING

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STATEMENT

How long does it take an ant to build a tunnel? That seems like a reasonable question. If you ever had an ant colony purchased by a well-meaning aunt for you in grade school you may have watched ants building industriously and you just might have an idea on this. To answer the question we might need some narrowing of scope, some simplification, and certainly some identification of terms and variables before we can get a nice answer. Let us identify some variables and then together make some assumptions which will lead to a mathematical model.

Let x be the length of the tunnel in feet that an ant builds.

Let $T(x)$ be the time in hours it takes the ant to build the tunnel of length x.

We can get some idea of our situation by making a sketch in Figure 1.

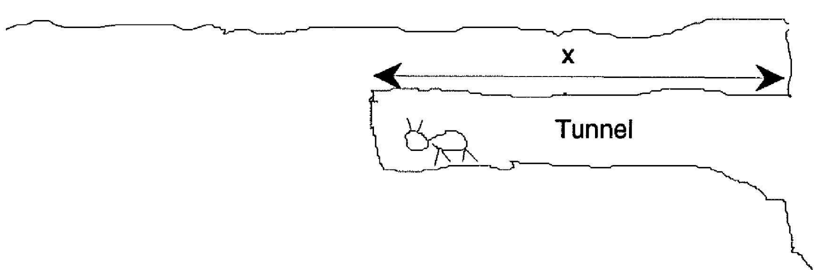

Figure 1. Crude drawing for ant tunnel building model. x is the length of the tunnel and $T(x)$ is the time it takes an ant to build a tunnel of length x.

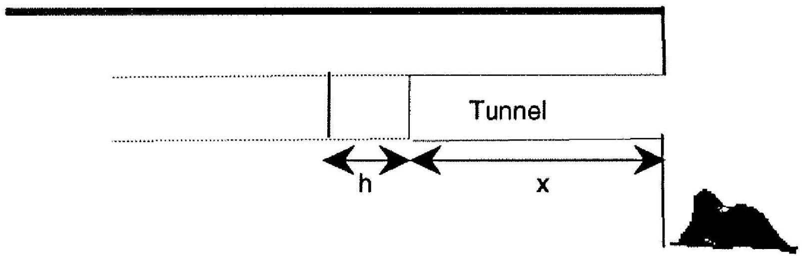

Figure 2. Useful diagram for discovering the time it takes to build a small section of the ant tunnel from distance x to $x+h$.

a) Write down several candidate functions for $T(x)$ and give one or two statements in each's defense and one or two statements against each.

We see that attempting to jump right on top of $T(x)$ can be hard. So, instead of going after $T(x)$ directly let us examine Figure 2. Consider some assumptions which (i) reflect the reality of such a situation and (ii) might make the model simple in a first attempt. Write these assumptions down and put the most useful ones as a prologue to your responses to (b) - (c).

Now see if we can build a model using these assumptions to help us tell how long it might take an ant to *extend* a tunnel from distance x to distance $x+h$. Thus we seek an expression for (1):

$$T(x+h) - T(x) = \underline{\hspace{4cm}} \tag{1}$$

Notice that $T(x+h) - T(x) \neq T(h)$, for $T(h)$ represents the time it takes to dig a small tunnel of length h from the mouth of the tunnel, while $T(x+h)-T(x)$ includes the time it takes to extend the tunnel from length x to $x+h$. This latter time must account for the time for the ant to bring the material all they way out along a path of length x from the region from x to $x+h$ which is more than just the time it takes, $T(h)$, to bring the material a distance of only h to the mouth of the tunnel.

b) List the variables (present or to be introduced) on which the expression in (1) might depend.

c) Below are several possible mathematical models for (1). Defend or reject each and offer your reasons. Perhaps modify one or two and make it better. When trying to reject a model consider some trivial cases and see if it makes sense, e.g., $h = 0$ or $x = 0$ or either h or x very large.

   i) $T(x+h) - T(x) = x + h$.

   ii) $T(x+h) - T(x) = x - h$.

   iii) $T(x+h) - T(x) = x^h$.

   iv) $T(x+h) - T(x) = x \cdot h$.

   v) $T(x+h) - T(x) = h^x$.

vi) $T(x+h) - T(x) = c$.

d) Convert your model difference equation (1) to a differential equation with appropriate initial conditions.

e) Solve the differential equation you create in (d) for $T(x)$. Hint: What initial condition $T(0)$ will you use?

f) Use your solution from (e) to determine how much longer it takes to build a tunnel which is twice as long as an original tunnel of length L. What would some of your original function models you set forth in (a) have told you here?

g) Suppose we had two ants digging from either side of our sand hill along the same straight line. How would this alter the total time for digging the tunnel?

Of course, we can apply these same principles of our model to real tunnel building for engineers.

h) If we were considering (g) as related to engineering construction of a long tunnel of length L, outline some of the issues we should be aware of when having two crews (one from each end of the tunnel) working on the tunnel.