

STUDENT VERSION

MIXING IT UP

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STATEMENT

Activity 1:

At time $t = 0$ a tank contains $Q(0) = 4$ lb of salt dissolved in 100 gal of water. Water containing 0.25 lb of salt per gallon is entering the tank at a rate of 3 gal/min, and the well-stirred solution leaves the tank at the same rate.

- Build a differential equation for the amount of salt, $Q(t)$, in lb in the tank at time t in min.
Hint: Keep track of the amount of salt that enters and exits the tank per minute.
- Find an expression for the amount of salt, $Q(t)$, in lb in the tank at time t in min and plot $Q(t)$ vs. t over time interval $[0, 200]$ min.
- Determine when the amount of salt doubles from the original amount in the tank.
- Determine when the amount of salt in the tank is 20 lb.
- Determine when the amount of salt in the tank is 30 lb.
- Determine the maximum amount of salt in the tank and when it occurs.
- Describe the long term behavior of the amount of salt in the tank using accompanying plots to support your description.

Activity 2:

At time $t = 0$ a tank contains $Q(0) = 4$ lb of salt dissolved in 100 gal of water. Water containing 0.25 lb of salt per gallon is entering the tank at a rate of 3 gal/min, and the well-stirred solution leaves the tank at a rate of 3.5 gal/min.

- a) Build a differential equation for the amount of salt, $Q(t)$, in lb in the tank at time t in min in this tank. Hint: Keep track of the amount of salt that enters and exits the tank per minute.
- b) Find an expression for the amount of salt, $Q(t)$, in lb in the tank at time t and plot $Q(t)$ vs. t over time interval $[0, 200]$ min.
- c) Determine when the amount of salt doubles from the original amount in the tank.
- d) Determine when the amount of salt in the tank is 20 lb.
- e) Determine when the amount of salt in the tank is 30 lb.
- f) Determine the maximum amount of salt in the tank and when it occurs.
- g) Describe the long term behavior of the amount of salt in the tank using accompanying plots to support your description.

Activity 3:

A 1200 gal tank has 400 gal of water in it containing exactly 20 lb of salt at the start of the process.

Salt water is being added to the tank. The concentration of the salt in the water being added is 0.3 lb/gal. This salt water mixture initially is added at a rate of 0.1 gal/min and this rate is steadily increased by 0.02 gal/min each minute.

The water in the tank is presumed to mix instantaneously and this mixture is being pumped out of the tank at the rate of 5 gal/min.

Let $Q(t)$ be the amount of salt in lb in the tank at time t in min.

- a) Describe the rate at which the salt water is being added to the tank as a function of time t in min.
- b) Build a differential equation for the amount of salt, $Q(t)$, in lb in the tank at time t in min in this tank. Hint: Keep track of the amount of salt that enters and exits the tank per minute.
- c) Determine the amount of salt, $Q(t)$, in lb in the tank at time t for the first 200 minutes of operation.
- d) Plot the amount of salt in the tank as a function of time t in the time interval $[0, 200]$ min. Explain what is going on in terms of the plot.
- e) What is the maximum amount of salt in the tank during the first 200 min of operation and when does this occur?
- f) Write a paragraph describing what happens to the salt and the water in the tank over the time interval $[0, 420]$ min. Offer plots to support your narrative.