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STUDENT VERSION

Drone Package Delivery

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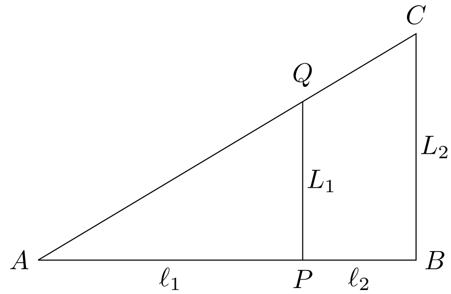
STATEMENT

You just started a new job with Amazon™! As a newbie, you are to practice delivering packages to houses with drones. So, you start with your own house! Your friend Jimbo volunteers to take the drone 5 miles east of your home, with the goal of flying it directly back to your home. The drone is $12.6 \times 12.6 \times 4.2$ (all in inches) in size. You program the black drone to always head directly to your house at a speed of **b** mph. Your house is located at the corner of 8$^{\text{th}}$ Ave. and 19$^{\text{th}}$ St. in New York City. There is a wind blowing from the south at a steady **w** mph. Assume the drone always flies at the same height. What is the flight path of the drone? Can you use numerical analysis to estimate the solution trajectory? What methods can you use?

Appendix: Calculus Review

First some geometry review! Recall that the two triangles ABC and APQ



are similar if the ratio of their corresponding sides are equal (in the above picture, they are!). That is,

$$\frac{L_1}{\ell_1} = \frac{L_2}{\ell_1 + \ell_2}, \quad \frac{L_2}{L_1} = \frac{\ell_1 + \ell_2}{\ell_1}.$$

Recall that a **parametric curve** $\mathcal{C}$ is a two dimensional curve that can be described by the equations

$$(x, y) = (f(t), g(t)).$$

If f, g are differentiable functions of t, and also y is differentiable is a function of x, then the chain rule says that

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt},$$

which means that

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}.$$

The value of $\dfrac{dy}{dx}$ gives the slope of the tangent to $\mathcal{C}$ at any point.

Example 0.1. *What shape is the curve given parametrically by* $(x, y) = (\sin^2 t, 2\cos(t))$*? HINT: How can you get to the familiar trigonometric identity* $\sin^2(t) + \cos^2(t) = 1$*?* ***ANS****: A parabola opening to the left, given by* $x = 1 - y^2/4$.

Example 0.2. *Does the parametric curve* $(x, y) = (t^2 - 2t, t^3 - 3t)$ *have a vertical tangent? HINT: consider* dy/dx*...* ***ANS****: No; notice that* $\frac{dy}{dx} = \frac{3t^2-3}{2t-2}$*. A vertical tangent would occur when* $2t-2 = 0$ *and* $3t^2 - 3 \neq 0$*; this does not happen.*

In $3D$, a parametric curve takes the form $\mathbf{r}(t) = (x(t), y(t), z(t))$. A parametric surface in $3D$ is of the form $\mathbf{r}(u, v) = (x(u, v), y(u, v), z(u, v))$. Recall that given a velocity vector $\mathbf{v}$ in $2D$ or $3D$, the speed of an object with such velocity vector is given by the magnitude of this vector:

$$\text{speed} = ||\mathbf{v}|| = \sqrt{x^2 + y^2 + z^2}$$

if $\mathbf{v} = (x, y, z)$. Now, if a particle's position is given by $\mathbf{r}(t) = (x(t), y(t), z(t))$, its velocity is $\mathbf{v}(t) = (x'(t), y'(t), z'(t))$.