SIMIODE - SCUDEM III 2018
Math Modeling Challenge

*Problem C - Snakes in the Long Run*

Executive Summary

Written By: Jon Mestas, Matthew Rue & Garrett Chavez
**Introduction**

Our team chose *Snakes in the Long Run* problem to model for the 2018 SCUDEM III competition. The problem is based on a study published in 1988 that relates the incubation temperature of reptile eggs, specifically pine snakes, to the male to female population ratio of the clutch or group of pine snake eggs (Burger, Joanna and R. T. Zappalorti. 1988.). The researchers concluded that sex ratio is a linear function of the incubation temperature. The relationship is as follows: $sexratio = 0.068 \times (Temp) - 0.95$. Over the past decade, effects of climate change has captured the attention of scientists around the world. One prediction about climate change is that there will be larger differences in yearly temperatures. Given the relationship between the pine snake sex ratio and temperature, two questions arise regarding the effects of climate change on the population dynamics: 1. Will the wider variations in temperatures impact the overall population dynamics of the Pine Snake? & 2. How long will it take for impacts, if any, to become noticeable?

**Background**

The study regarding pine snake sex ratio was conducted from 1976 to 1986 in three New Jersey counties: Ocean, Atlantic & Burlington. These counties are located in the Southern area of the state, two of the counties (Atlantic & Burlington) share boundaries with Wharton State Forest which is one of the largest state forests (122,880 acres) within the three counties observed in the pine snake sex ratio study. Since the ratio is dependant on temperature, our team decided to model New Jersey’s historical temperature trend in an effort to forecast the future pine snake sex ratio. Particularly, we focused on the Wharton State Forest area, making the assumption that the state forest would be prime habitat for the majority of the New Jersey’s pine snake population. While the historical weather data was unavailable for the state park itself, we did find historical weather data for the town of Hammonton, NJ which is located along the Southwest border of Wharton State Forest. Due to the close proximity of Hammonton to the state forest, we gathered our weather for this town to help create our temperature model. To produce a more accurate model weather data from neighboring town would be of benefit, however it was time prohibitive to gather 42 years of weather data for multiple towns in light of the competition deadline.

**Model**

We collected the average temperature for the month of August for the town of Hammonton from 1976 to 2018 in increments of 2 years. August typically concludes the mating season for the pine snake, such the reason for our choice of month. Our objective was to plot the historical weather data and use the trend to roughly forecast the temperature in the future and in turn the sex ratio. Initially, we plotted our data without a trendline to observe the patterns without bias of a particular function. Upon examination of the data plot it appeared the average weather trend over the last 42 years appeared to be sinusoidal, however given the assumption that climate
change will cause greater differences in yearly temperatures, this would indicate that the
temperature (amplitude) may increase with time. Similar to the exponential dampening of a
spring in classical mechanics, which can be modeled with force equations and derived with
differential equations, except, in this case the temperature is not dampening.

The temperature is increasing with time, however modeling the temperature increase with
an exponential function would only be accurate over the initial part of the function. As time goes
to infinity, the temperature would also go toward infinity, making the model inaccurate in the
long run. Also, the period of the plotted sinusoidal data varied, this is something that we are
unsure how to model or even if can be modeled. A much more simpler approach that we
determined would be to model the average temperature trend with a simple linear function,
which is how we decided to model our question. This provided us with temperature as a function
as time in years, we then used this temperature function to define the sex ratio as a function of
time. Applying implicit differentiation to this equation reveals the rate at which the sex ratio is
changing with respect to time. Derivation in the Appendix.

Solution

Our calculated ratio; $\Delta R(t)=0.0438$ represent the change of the male to female ratio with
respect to time.

Based on our calculations, fluctuations in temperatures will have an overall impact on the
population dynamic of the pine snake. However, this would have to be observed over a extensive
period of time. According to our data, it would take approximately 114 years for us to see the sex
ratio began to favor the female population. And by our assumptions, an increase in the female
population will gradually increase the overall population due to a less competitive breeding
environment. However, this claim is an assumption and with other empirical data we may be
able to provide a more educated estimation.

Other forms of sampling may have been conducted to improve our data, or provide us
with a more accurate model. For our analysis, we chose to pick the location that was in the
closest proximity to Wharton State Forest. By limiting our data section to only one selected area,
we fail to take into consideration other snake locations in which a fluctuation in temperature
could be more drastic, therefore influencing our calculated ratio.

Conclusion

In conclusion, with more numerical data, we would have been able to generate a more
accurate curve of linearity to model our ratio. Modeling different locations, adding more time
increments, and assuming climate change modeling will stay consistent in the upcoming decades
would ultimately add an improvement to our ratio function.

We have concluded, by our estimations, that in roughly 114 years we will see a
significant dynamic change in the population of the Pine Snake with the ratio become nearly
inverted to favor female populations.
Appendix:

References

[1] NJDEP New Jersey Department of Environmental Protection, Accessed 10/26/18
www.state.nj.us/dep/parksandforests/parks/wharton.html.


Equations:

\[ R(T) = 0.068T - 0.95 \]
\[ T(t) = 0.0632t + 73.902 \]
\[ R(T(t)) = 0.068(0.0632t + 73.902) - 0.95 \]
\[ R(T(t)) = 0.0438t + 5.025 \]
\[ \frac{dR}{dt}(0.0438t + 5.025) \]
\[ \Delta R(t) = 0.0438 \]