Temperature Affect on Pine Snake Population

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Introduction

- A paper researched by Burger and Zappalorti suggests a linear dependence between incubation temperature (Celsius) and the sex ratio (male/female) of Pine snake.
  - \[ \text{SexRatio} \approx 0.068 \times T - 0.95 \]

- Sex ratio influences the reproduction rate of Pine snake, which alters population of Pine snake

- Environmental temperature affects population of snakes

- Differential equation to model the population dynamics of Pine snake based on temperature variation.
Modeling

- **Start: logistic function:** \[ \frac{dP}{dt} = k_1 P (C - P) \]
  - \( k_1 \): growth rate
  - \( C \): environment capacity
  - \( P \): total population

- **Considering death rate:**
  \[ \frac{dP}{dt} = k_1 P (C - P) - k_2 P \]
Modeling

- For birth, male : female = \( \frac{r}{1+r} : \frac{1}{1+r} \);
- For death, male : female = \( \frac{m}{m+f} : \frac{f}{m+f} \).

Therefore,

\[
\frac{dm}{dt} = \frac{r}{1+r} k_1 P(C - P) - k_2 \frac{m}{m+f} P
\]

\[
\frac{df}{dt} = \frac{1}{1+r} k_1 P(C - P) - k_2 \frac{f}{m+f} P
\]
Modeling

- Reproduction \[ \rightarrow \] Interaction \[ \rightarrow \] \( mf \)

- General knowledge
  - female is more important
  - \( mf^x \) (\( x > 1 \))

- Our model evolves into:
  \[
  \frac{dm}{dt} = r \, \frac{k_1 mf^x P(C - P)}{1 + r} - k_2 \, \frac{m}{m + f} P
  \]
  \[
  \frac{df}{dt} = 1 + r \, \frac{k_1 mf^x P(C - P)}{1 + r} - k_2 \, \frac{f}{m + f} P
  \]
Modeling

- Important factors has been considered, but...

**Balance!!!**

- “Normalization”

- Scaled into [0,2]
  - divide $(C - P)$ by $C/2$
  - divide $m$ by $P/2$ and $f^x$ by $\left(\frac{P}{2}\right)^x$
Modeling

- After “factor-scaled” and simplifying equation...

- Our final, elegant model:

\[
\begin{align*}
\frac{dm}{dt} &= \left[ k_1 \frac{r}{r+1} \frac{mf^x}{(m+f)^{x-1}} \left(1 - \frac{m+f}{C}\right)^{2x+2}\right] - k_2 m \\
\frac{df}{dt} &= \left[ k_1 \frac{1}{r+1} \frac{mf^x}{(m+f)^{x-1}} \left(1 - \frac{m+f}{C}\right)^{2x+2}\right] - k_2 f
\end{align*}
\]
Simulation

- Temperature equation:
  \[ T = (avg + q_1 t) + var(q_2 t)\sin(0.2\pi t) \]
  - With average temperature increase \( q_1 \) per year and variation of temperature increase \( q_2 \) per year

- We simulate our model in four case:
  1. Constant average temperature and fluctuation increase 2.5% per year
  2. Average temperature increase 0.1 °C and no fluctuation each year
  3. Average temperature increase 0.1 °C and fluctuation increase 2.5% per year
  4. Average temperature decrease 0.1 °C and fluctuation increase 2.5% per year

- Initialization:
  - Population model: \( f = m = 100, C = 500, k_1 = 0.6, k_2 = 0.4, x = 3 \)
  - Temperature equation: \( avg = 28.6°C, var = 2 \)
Simulation Graph

Population of Snake
with average temperature increasing 0 °C per year
and fluctuation increasing 2.5 % per year

- Population fluctuates with temperature.
- No increase or decrease trend
Simulation Graph

Population of Snake with average temperature increasing 0.1 °C per year and fluctuation increasing 0% per year

- Total decrease of snake population over time
- Female decreases sharper than male
Simulation Graph

Population of Snake
with average temperature increasing 0.1 °C per year
and fluctuation increasing 2.5 % per year

- Most realistic assumption
- Combination of two factors
Simulation Graph

Population of Snake
with average temperature decreasing 0.1 °C per year
and fluctuation increasing 2.5% per year

- Female increase
- Male decrease
- Total population increase
  - But have a limit
  - Even a downward trend
Evolutionary Pressures

- The population of snakes will decrease because the mating and egg hatching time increase each year
- How can they adapt?
  - Can change their mating time
    - Different mating season
      - For example, summer to spring
Effect of Evolutionary Pressures
Changing Mating Season

- Population decreases initially
- Increases after 50 years as the snakes find a new mating season
  - Percentage of female increases as the hatching temperature decreases from finding a new mating season
Conclusion

- Climate variation causes population fluctuation
- Global warming causes the population to decrease
  - Have a combined influence
- Temperature decreasing could cause population to increase
  - Has a limit

- Higher birth rate...
  - Increase the population faster initially, effect seen sooner
- Higher capacity...
  - Slower to reach limit, population fluctuates at higher rate
- Higher temperature...
  - Population will decrease faster
- Higher variation...
  - Effect will be significant with fluctuating temperature
Limitation

- Temperature
  - Stochastic process, Markov chain

- Biology and Geography influence
  - Many factors can be affected by temperature

- Capacity
  - Non-linear may be better

Thank You for Listening
Questions ???