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STUDENT VERSION
Three Hole Column

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STATEMENT

Consider a cylinder containing a falling column of water as water exits through three spigots (see Figure 1).

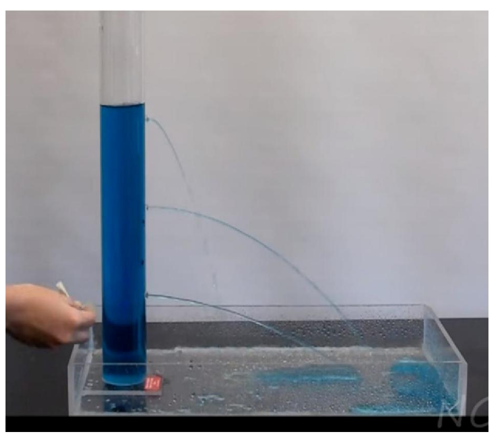

Figure 1. Cylinder with column of water and three spigots.[1]

This cylinder has three exit holes or spigots. We create a cylinder and offer dimensions, perhaps different from the one depicted in [1], and ask you to model the height of the water in the cylindrical column as a function of time using Torricelli's Law[3]. Wewill consider two cases. First we will vary the radii of the spigot holes and second we will keep the radii of the spigot holes the same.

Case 1 - varied radii of the spigot holes

The radius of the cylinder is $R = 2.5$ cm and the radius of the lower spigot hole is $r_3 = 0.8$ cm, the radius of the middle spigot is $r_2 = 0.5$ cm, and the radius of the top spigot hole is $r_1 = 0.2$ cm.

The middle spigot hole is 12 cm above the bottom spigot hole (measured from center of the hole to center of hole) while the top the spigot hole is 10 cm above the middle spigot whole (measured from center of the spigot hole to center of the spigot hole).

Identify the spigot holes as bottom is spigot hole 3, middle spigot hole is hole 2, and top spigot hole is hole 1.

The water level initially is at $h_0 k = 14$ cm above the top spigot hole (measured from center of the spigot hole to the water level). The top spigot hole is $h_1 k = 10$ cm above the middle spigot hole (measured from center of the spigot hole to center of the other spigot hole) while the middle spigot hole is $h_2 k = 12$ cm above the bottom spigot hole (measured from center of the spigot hole to center of the other spigot hole). The bottom spigot hole is $h_3 k = 5$ cm above the base of the container.

Case 2 - identical radii of the spigot holes

The radius of the cylinder is $R = 2.5$ cm and the radius of the lower spigot hole is $r_3 = 0.3$ cm, the radius of the middle spigot hole is $r_2 = 0.3$ cm, and the radius of the top spigot hole is $r_1 = 0.3$ cm.

The middle spigot hole is 12 cm above the bottom spigot hole (measured from center of the spigot hole to center of spigot hole) while the top spigot hole is 10 cm above the middle spigot hole (measured from center of the spigot hole to center of the spigot hole).

Identify the spigot holes as bottom is spigot hole 3, middle spigot hole is spigot hole 2, and top spigot hole is spigot hole 1.

The water level initially is at $h_0 k = 14$ cm above the top spigot hole (measured from center of the spigot hole to the water level). The top spigot hole is $h_1 k = 10$ cm above the middle spigot whole (measured from center of the spigot hole to center of the other spigot hole) while the middle spigot hole is $h_2 k = 12$ cm above the bottom spigot hole (measured from center of the spigot hole to center of the other spigot hole). The bottom spigot hole is $h_3 k = 5$ cm above the base of the container.

Analysis

Recall from [3] that the exit velocity of the stream of water exiting a column of water at a depth of h units below the surface of the column of water is given by $v = \sqrt{2gh}$, where g is the acceleration due to gravity in the appropriate units.

Using this fact you should be able to build a set of differential equations for the falling column of water when the surface is in the three regions: (i) above spigot 1, (ii) between spigot 1 and spigot 2, and (iii) below spigot 2 and above spigot 3. In each case you can compute the rate at which the volume is changing from the outflow of the spigots and set that equal to the volume loss, $\pi R^2 h'(t)$

where $h(t)$ is the height of the column of water above a spigot hole, $h'(t)$ is the rate at which the height of the column of water is falling, and πR^2 is the constant cross sectional area of the cylinder.

One other thing to consider is the fact that through any spigot hole about only 70% of the possible flow actually occurs due to friction on the spigot wall as an example of loss. So we need to employ some lessening of the flow rate by multiplying the theoretical or maximum flow rate by some constant $\alpha = 0.70$, where α is an empirically observed number which indicates the percentage of the maximum flow rate which actually gets through the small spigot hole. α is called *the discharge or contraction coefficient*[2].

You should produce plots of the height of the column of water over the several sections of the column of water above and between spigots and in Case 2 with identical radii of the spigot holes you should comment on the rate at which the height of the column of water decreases in each section.

REFERENCES

[1] North Carolina State University, Physics Department DemoRoom. 2018. *Velocity of Efflux Three Hole Can Experiment.*. Raleigh NC: North Carolina State University DemoRoom. `https://www.youtube.com/watch?v=0EsUiBhBIX8`. Accessed 15 December 2018.

[2] Wikipedia contributors. 2018. Discharge coefficient. Wikipedia, The Free Encyclopedia. `https://en.wikipedia.org/w/index.php?title=Discharge_coefficient&oldid=839574186`. Accessed 17 December 2018.

[3] Winkel, B. 2015. 1-015-S-Torricelli. `https://www.simiode.org/resources/488`. Accessed 15 December 2018.