SCUDEM III 2018 - Author Commentary on Problems and Submissions

Kelly Black
Department of Mathematics
University of Georgia

1 Overview

The Fall 2018 SIMIODE Challenge Using Differential Equations Modeling (SCUDEM III 2018) featured three problems. Some observations of the work submitted by the student teams is given here. It is assumed that the reader is familiar with the three problems which are available on the SIMIODE website.

The teams were asked to address one of the problems, and each student team was asked to submit an executive summary of their work as well as the slides they used for a 10 minute presentation on Challenge Saturday, 27 October 2018, at a local site near their home school.

As author of the problems my observations of the submitted work is broken up into two sections. The focus of the first section is the executive summary. The focus of the second section is the material associated with the presentations. Within each section some general observations are given for each of the three problems.

For each section the comments about the student entries are given in the same order. The first problem examined is the set of questions associated with conflict between patrilineal clans. The second problem is the set of questions on a swing used in a museum exhibit. The final problem come from the set of questions about the role of temperature and sex determination in Pine snakes.

2 Executive Summaries

The executive summaries continue to be written in a wide range of styles. Some of the summaries are terse and may include some equations or a brief discussion of a team's results. This year a larger percentage of the summaries seemed to be more complete and included an abstract, an overview of the problem, a brief discussion of a model with motivations, and a brief discussion of results or predictions. This is good!

There is no one format for an executive summary. It is important, though, that someone should be able to read the executive summary and get a feel for a team's approach, motivations, and results. It is also important that an executive summary be easily read and understood by someone who is not familiar with the original problem statement.

2.1 Conflict Between Patrilineal Clans

The problem about conflict between clans is motivated by a paper that discusses one possible explanation for an observation about the lack of diversity of people carrying a Y chromosome. A theory put forward about an observed lack of variation is that roughly 7,000 years ago conflict between different human groups was responsible for a decline in the male population, but the impact on the female population was less severe. The problem required teams to assume the theory is correct and construct mathematical models of conflict between groups and observe what the impact is on the resulting sex ratio for the human population.

A large number of groups chose a model that is based on a Lotka-Volterra model. Many of the resulting models were composed of systems with four populations. The basic idea is that two groups are in conflict, and each group has two populations, male and female.

For many of the executive summaries the system of equations was stated, but it was not always clear how the conflict between the groups give rise to the given model. Only a small number of papers discussed the motivation for individual terms or how the dynamics gave rise to the given terms in the model. For example, if the population density of people carrying a Y chromosome for clan one is denoted $M_1(t)$, some student groups included a term for this clan of the form $-\alpha M_1^2(t)$, where α is a constant. It is not clear why this kind of penalty term is appropriate for a group of related hominids. Good modeling merits a rationale for terms.

Some groups provided more details and tended to make a better impression. For example, some groups restated the problem and did not assume that the reader was familiar with the original problem. Such entries tended to be easier to read and included more background material that also helped motivate the group's work.

One thing that was notable, is that the system of equations that arise from a Lotka-Volterra approach tend to be more amenable to determining the fixed points of the system. Unfortunately, this was discussed in only a small number of the summaries. An examination of the fixed points and whether or not a model will tend to get close to and stay near a fixed point is an important consideration in any model.

2.2 Museum Exhibit

The question about a museum exhibit provides a constrained motion that can be difficult to describe. Overall, the teams did a good job of describing the resulting motion, and the teams tended to do a better job of discussing the given problem as compared to the question on conflict between clans. Like the question on clans, there were a number of summaries which provided an equation of motion, but did not provide insights into the motivation or meaning of the various terms within the model. This is important to do in modeling!

The summaries for the museum exhibit question tended to provide more in the way of results compared to the question on clans. It was not always clear how the results were obtained. The teams tended to provide the results in written form rather than render a graphical view of the results. For example, a view of the path of the mass was rarely given and few results were given with respect to a phase plane.

This particular problem was complicated with respect to the apparatus to examine. However, the basic equations of motion can be derived using Newton's Second Law. Some teams tried other approaches based on physical principles that result from Newton's Second Law, but it was not always clear how those models could provide the temporal dynamics associated with the questions.

Another aspect of the summaries is that different teams tended to approach the issue of assumptions in quite different ways. Some teams omitted the assumptions, some teams provided a bulleted list, and other teams provided details with explanations and motivations. It can be difficult to decide how many details to include in a summary, and a number of teams did an excellent job of providing just enough details to balance brevity with the details necessary to describe their model. This is an admirable goal.

Finally, the use of graphs and figures to inform a reader is an important topic, and it is a vital technique when providing a summary. A figure can quickly drive home a point and focus attention on an aspect that the team feels is important. For a set of questions focused on the dynamics of a moving object the construction of a free-body diagram is a vital step. Several teams did an excellent job of creating free-body diagrams and incorporating them into their summaries.

2.3 Sex Ratios of Snakes

The final problem in the event required teams to examine the dynamics of a single population of snakes, and the ratio of male to female snakes varies with respect to the eggs' incubation temperature. This problem attracted the largest number of entries, and the entries associated with this question tended to be more complete with respect to the various issues raised above.

As noted above, some of the summaries provided a model without sufficient motivation and discussion to fully inform the reader as to why the model captures the important dynamics associated with the questions. A number of other teams, though, provided more discussion about their models. Also, the summaries for this question were more likely to include an overview of the problem itself, and these summaries tended to better stand on their own.

Additionally, the teams tended to provide more details about their results in this problem. I saw more summaries that included a written overview of the dynamics as well as graphical views of the results of the models. Teams that tied together their description with the figures were the easiest to read and understand. Additionally, the teams tended to do a good job in labeling and annotating their plots which helps immensely in trying to understand the teams' results.

3 Presentations

There are many ways to present and discuss abstract ideas. In the first phase of the event teams are asked to discuss their ideas in a brief written form, an executive summary. In the second phase the teams are asked to discuss their ideas in a verbal form through a 10 minute presentation. The two forms require different approaches and require different communication skills. In this section I will provide some observations of the presentations made by the groups. Of course I only examined the presentation material and were not present at the presentations themselves. The observations will be given in the same order as the previous section.

3.1 Conflict Between Patrilineal Clans

The first problem, modeling demographic trends between clans, required student teams to decide on how the two clans interact. A team's basic assumptions directly impact the modeling choices. For any modeling question the assumptions are important. The way the teams discussed their basic assumptions varied widely. Fortunately, most teams explicitly discussed their assumptions while the amount of details varied.

Another important aspect of a model is to take a critical look at the model itself. Modeling a situation does not end with the first model, and it generally requires multiple iterations before a mathematical model has been determined. Before a second look at a model can take place a team should explicitly identify the strengths as well as the weaknesses of a model. The teams that took this extra step demonstrated one of the most important parts of the modeling process.

Once the relative strengths and weaknesses are identified, the next step in the process is to identify what updates should be made. A small number of teams stated which parts of the model they would like to change in a future update to their model. This requires the teams to take a critical look at their model's strengths and weaknesses. This examination also requires them to decide which parts of the model require the greatest amount of attention.

The idea of iteration and updating a model is a central idea. This part of the process also includes interactions with other researchers. It also means delving deeper into the underlying interactions. There is an obligation to fully disclose the sources and inspiration for different ideas. Many teams did a great job of including a list of references, and it was noteworthy that the presentations seemed to be more likely to include references. The teams should go further, though, and include citations within the presentation of the ideas as well.

As a final note, it was interesting to see that many of the teams tended to provide detailed motivation for the problem and the model itself in their presentations. They did not assume that everyone in the audience was aware of the original problem statement. In doing so they created a context and insured that everybody was aware of the relevant questions.

3.2 Museum Exhibit

The second problem, the museum exhibit, required the construction of a model based on fundamental physical principles. This requires that the system be fully identified and described prior to the derivation of the model. Compared to the executive summaries, the teams tended to do a better job of describing the system in their presentations. In particular, it was quite common to see free body diagrams and more careful descriptions of the apparatus. (At least one team built their own version of the apparatus!) The inclusion of the diagrams was quite helpful in understanding the decisions the teams made to construct their models.

Related to the graphical representation of the object, more teams also appeared to include different graphical representations of the solutions. In particular, many teams included graphs of the phase plane representations of their approximations. Many also included the time varying solutions of their approximations. Being able to see multiple ways to visualize the approximations made it much easier to understand the results and better see the extent of their excellent work.

In terms of the modeling process, one aspect of the teams' work stood out compared to the first problem. It seemed to be more likely that the teams reflected on their process. This is an excellent practice in order to determine how to proceed in the following iterations. If a team can identify the strengths and weaknesses about how they conducted their modeling effort, then they can improve on their methodology which hopefully translates into a more efficient modeling process.

The final aspect of note is how some of the teams conducted the analysis of their model. One important question about modeling is what happens when there is a small change in a parameter. A number of teams examined what would happen in their model if they changed one parameter. They did so by looking at the results over a range of values, and many teams did an excellent job of presenting their results. In doing so they were able to identify the aspects of the design that are most sensitive to changes or errors, and it is a direct aid to understanding what are the most important aspects of the system to better understand.

3.3 Sex Ratios of Snakes

The presentations for the final question, the sex ratio of a population of snakes, had many of the features discussed for the previous two problems, and some of the aspects discussed here are similar to topics already discussed. For example, it has been noted that teams were more careful to discuss the questions and context in their presentations. In addition to discussing the problem, many of the presentations for the third question also included more details about the terminology and about interactions. For people not familiar with the context this helps reduce potential confusion when later discussing the model.

A number of student teams also included nice graphs depicting the various interactions the teams decided were important. These graphs make it much easier to understand the complex interactions between the different groups under consideration. In this case, the resulting models depart from many mathematical models which tend not to treat different sexes as different populations, and the graphical view of the interactions helped tremendously when examining the resulting models.

As an example, many student teams discussed their assumptions. A list of assumptions can be long, and the particular interactions that result from those assumptions are not immediately obvious. A simple graph can immediately demonstrate the important interactions and groups, and it is easier to understand which groups the interactions impact. When viewing the resulting model it becomes much easier to understand the origin of particular terms within the model.

Another important topic previously discussed is the sensitivity of a model. Some of the student teams discussed what happens for a small change in a given parameter for this question. This is particularly important given that the regression lines are the result of a statistical analysis, and a true slope is not known. In this case it is even more important to see what happens for small changes in the model.

One impressive part of the presentations submitted by several teams is that some of the teams also looked at what happens with respect to changes in the assumptions about the system. This is a relatively sophisticated approach and deserves praise for the teams who took on this difficult analysis. For example, some teams made assumptions about how temperatures might vary in the future, and they looked at how their results changed when their assumption about future variation differed.

For the third problem, the variation in the temperatures is central to the questions posed. Many of the predictions about future climate are difficult to assess given that the predictions indicate that one of the main differences will be greater variations in the predicted trends. A number of teams did an excellent job in directly examining this part of the problem and presented the results of their model under different assumptions about future temperature variations.

With respect to introspection of the model and looking at the strengths and weaknesses of a model, there was not a lot of attention paid to the validity of the model under extremes. Most mathematical models provide important insights, but only within a limited range of the assumptions. For example, some teams examined the results of their model for very high and very low temperatures. At some point, though, the model itself is not valid beyond some given threshold. A more careful analysis of the approach itself would be helpful in understanding the possible circumstances where the model results might be appropriate.

Finally, the presentations for the third question also tended to include a better set of references used by the teams. As noted above the presentations often included references, but few also included citations. Without knowing which ideas in the presentation were influenced by the sources it is difficult to figure out the references were used and how the teams made use of the various ideas.