

STUDENT VERSION

SHAMPOO

STATEMENT

Our shampoo has a cap which we must unscrew and then set aside both bottle and cap in our shower hanging dish, thus leaving the shampoo bottle open for shower water to enter. In fact, we unscrew the top, set the top aside and then do two washes of our hair. We use one “portion” of shampoo for a first wash of our hair and a second equal “portion” of shampoo for the second wash of our hair.

Between the first and second wash when we set the shampoo bottle aside in our shower hanging dish a slight amount of water enters the shampoo bottle through the open top of the shampoo bottle. Some mixes in with the shampoo, but most sits in the top of the shampoo bottle. In our second wash some of that water comes out with the shampoo, reducing the concentration of shampoo ingredients we apply to our hair. After the second wash we set the bottle on our shower hanging dish until we are finished with our shower. At that time we recap the shampoo bottle, but in the meantime from the end of our second wash until we recap the shampoo bottle the shower water does enter the shampoo bottle. How much? Well it depends upon the length of our shower, the opening size of the shampoo bottle, the placement of the shower hanging dish, etc. Other considerations?

So let us say that for each time we shower in this manner we actually add a fixed amount of shower water to the shampoo and because of shaking, perhaps falling from its perch on our shower hanging dish, or some diffusion we will assume the water is mixed thoroughly with the shampoo by the time we take another shower.

1. Model the amount of shampoo in our shampoo bottle at time t . Hint: Build a model based on the change of the amount of shampoo over time.

After you have modeled the shampoo problem then ask questions of it.

2. How long will the shampoo last and what will be the shampoo amount and concentration over time in the bottle?

3. When is the amount of water greatest in the shampoo bottle and just how much water is in the bottle at that time?
4. When is the concentration of shampoo lowest in the shampoo bottle and just how much water is in the bottle at that time?
5. Modify the scenario so that the shower taker realizes he/she has been diluting the water and so remembers for a time to put the lid on immediately for a week or so. Maybe this happens every other week, or is noticeable when the bottle is, for example, exactly half - full. Address the questions (2) - (4) in this case.
6. Model this using a difference equation approach if you used a differential equation approach or vice-versa and then compare your model predictions.